

Some solutions for
the Exercises sheet from
27/11/2006

Exercise 2):

1. Transform the following formulae to clausal form using the three steps from the lecture and write down the resulting form as a set of clauses:

- $\forall x \neg \exists y \forall z (P(h(z)) \rightarrow \neg Q(x, y))$

- Step 1a: Variablenames are not conflicting.

- Step 1b: $\forall x \forall y \exists z (P(h(z)) \wedge Q(x, y))$

- Step 2: $\forall x \forall y (P(h(f(y, x))) \wedge Q(x, y))$

- Step 3: The formula is already in CNF!

$$\Rightarrow \{P(h(f(y, x))) \leftarrow, Q(x, y) \leftarrow\}$$

- $\neg \forall x \exists y (Q(x, f(y)) \wedge Q(x, d) \rightarrow \neg R(x, g(y, f(z))))$

Variable z is not quantified, so the formula is not closed, but: A formula F with free variables x is satisfiable iff $\exists x F$ is satisfiable.

- Step 1a: $\exists z \neg \forall x \exists y (Q(x, f(y)) \wedge Q(x, d) \rightarrow \neg R(x, g(y, f(z))))$

- Step 1b: $\exists z \exists x \forall y ((\neg Q(x, f(y)) \vee \neg Q(x, d)) \wedge R(x, g(y, f(z))))$

- Step 2: $\forall y ((\neg Q(a, f(y)) \vee \neg Q(a, d)) \wedge R(a, g(y, f(b))))$

- Step 3: The formula is already in CNF!

$$\Rightarrow \{\leftarrow Q(a, f(y)), \neg Q(a, d) \quad R(a, g(y, f(b))) \leftarrow\}$$

$$\forall(Q(x) \vee ((R(a, x) \wedge P(y)) \vee (P(b) \wedge Q(b))))$$

Formula is already in form after Step 2.

After Step 3:

$$\begin{aligned} &\forall(Q(x) \vee R(a, x) \vee P(b)) \wedge \\ &(Q(x) \vee P(y) \vee P(b)) \wedge \\ &(Q(x) \vee R(a, x) \vee Q(b)) \wedge \\ &(Q(x) \vee P(y) \vee Q(b)) \end{aligned}$$

$$\Rightarrow \left\{ \begin{array}{l} Q(x), R(a, x), P(b) \leftarrow \\ Q(x), P(y), P(b) \leftarrow \\ Q(x), R(a, x), Q(b) \leftarrow \\ Q(x), P(y), Q(b) \leftarrow \end{array} \right\}$$

Exercise 3)

Step 1b:

$$A = \neg((\forall x P(a, b, x) \wedge \forall u \forall v \exists w (P(u, v, w) \rightarrow R(w)) \wedge \forall y (R(y) \rightarrow R(f(y)))) \rightarrow \exists z R(f(z)))$$

$$\Rightarrow \neg(\neg(\forall x P(a, b, x) \wedge \forall u \forall v \exists w (P(u, v, w) \rightarrow R(w)) \wedge \forall y (R(y) \rightarrow R(f(y)))) \vee \exists z R(f(z)))$$

$$\Rightarrow (\forall x P(a, b, x) \wedge \forall u \forall v \exists w (P(u, v, w) \rightarrow R(w)) \wedge \forall y (R(y) \rightarrow R(f(y)))) \wedge \neg \exists z R(f(z))$$

$$\Rightarrow (\forall x P(a, b, x) \wedge \forall u \forall v \exists w (P(u, v, w) \rightarrow R(w)) \wedge \forall y (\neg R(y) \vee R(f(y)))) \wedge \neg \exists z R(f(z))$$

$$\Rightarrow (\forall x P(a, b, x) \wedge \forall u \forall v \exists w (\neg P(u, v, w) \vee R(w)) \wedge \forall y (\neg R(y) \vee R(f(y)))) \wedge \neg \exists z R(f(z))$$

$$\Rightarrow (\forall x P(a, b, x) \wedge \forall u \forall v \exists w (\neg P(u, v, w) \vee R(w)) \wedge \forall y (\neg R(y) \vee R(f(y)))) \wedge \forall z \neg R(f(z))$$

Step 2:

$$\Rightarrow (\forall x P(a, b, x) \wedge \forall u \forall v (\neg P(u, v, g(u, v)) \vee R(g(u, v))) \wedge \forall y (\neg R(y) \vee R(f(y)))) \wedge \forall z \neg R(f(z))$$

Step 3: The formula is already in CNF!

$$S = \{ \begin{array}{l} P(a, b, x) \leftarrow \\ R(g(u, v)) \leftarrow P(u, v, g(u, v)) \\ R(f(y)) \leftarrow R(y) \\ \leftarrow R(f(z)) \end{array}$$

Refutation on the whiteboard...

Exercise 4)

Find an mgu for each of the following pairs

1. $(p(u, v), p(y, f(y)))$

2. $(p(a, x, f(g(y))), p(z, h(z, w), f(w)))$

3. $(q(f(x, g(y, x)), a, g(b, z)), q(f(v, w), u, g(b, a)))$

1. $\text{mgu} = \{u/y, v/f(y)\}$
2. $\text{mgu} = \{z/a, x/h(a, g(y)), w/g(y)\}$
3. $\text{mgu} = \{x/v, w/g(y, v), u/a, z/a\}$

Go through last example in detail...

Exercise sheet 5)

($P(u,u)$, $P(x,f(x))$)

$\theta_1 = \{u/x\}$

($P(x,x)$, $P(x,f(x))$)

Occur check fails for disagreement $(x,f(x))!$

If occ. check is omitted:

$\theta_2 = \theta_1\{x/f(x)\}$

($P(f(x),f(x))$, $P(f(x),f(f(x)))$)

...

Infinite loop!

Exercise sheet 2, 6)+7)

6)

$$\begin{aligned} & (P(x_1, x_2, \dots, x_n) \quad P(f(x_0, x_0), f(x_1, x_1), f(x_2, x_2) \dots f(x_{n+1}, x_{n+1}))) \\ \theta_1 &= \{x_1/f(x_0, x_0)\} \\ & (P(f(x_0, x_0), x_2, \dots, x_n) \quad P(f(x_0, x_0), f(f(x_0, x_0), f(x_0, x_0)), \dots f(x_{n+1}, x_{n+1}))) \\ \theta_2 &= \{x_1/f(x_0, x_0), x_2/f(f(x_0, x_0), f(x_0, x_0))\} \\ & \dots \end{aligned}$$

In the k-th iteration, we have an exponential number of function symbols f in the Q_{θ_k} , so the occur check will need exponential time!

7) Easy, check e.g. [Lloyd, 1987]