Exercises on Intuitionistic and Modal Logics

David Pearce

Universidad Rey Juan Carlos (Madrid, Spain)

1.

1. Prove by natural deduction the following theorems of intuitionistic logic H:

$$\neg\neg\neg\varphi \rightarrow \neg\varphi$$

$$(\varphi \rightarrow \psi) \rightarrow (\neg\psi \rightarrow \neg\varphi)$$

$$(\varphi \rightarrow \psi) \rightarrow (\neg\neg\varphi \rightarrow \neg\neg\psi)$$

2. Prove the "monotonicity"property for Kripke models for H, ie that for any model $\langle W, \leq, i\rangle$ and formula φ:

$$\varphi \in i(w) \text{ and } w \leq w' \Rightarrow \varphi \in i(w').$$

3. Show by counter-models that the following are not theorems of H (hint: use "fork"models):

$$\neg\neg\varphi \vee \neg\varphi$$

$$\neg(\varphi \wedge \psi) \rightarrow (\neg\varphi \vee \neg\psi)$$

.

4. Use Kripke models for the logic of here-and-there to show that the formulas of question 3 are theorems in here-and-there.
5. In constructive logic N with strong negation, show that $\neg\varphi =_{def} \varphi \rightarrow \sim\varphi$ is a correct definition. Prove the following:

$$\vdash \varphi \leftrightarrow \sim\sim\varphi$$

$$\vdash \sim(\varphi \wedge \psi) \leftrightarrow (\sim\varphi \vee \sim\psi)$$

$$\nvdash (\varphi \rightarrow \psi) \rightarrow (\sim\psi \rightarrow \sim\varphi)$$

6. 1. Prove completeness for $S4$ by showing that the canonical model is reflexive and transitive
   2. Prove completeness for B by showing that the canonical model is reflexive and symmetrical
   3. Prove completeness for D by showing that the canonical model has a serial accessibility relation
7. Check whether $(\alpha \rightarrow \beta) \rightarrow (\sim\beta \rightarrow \sim\alpha)$ is a theorem of here-and-there with strong negation, N_5. If not, provide a counter-model or counter-assignment.
8. Prove the "supportedness"property of disjunctive logic programs under answer set semantics, ie the property that for any program Π if a literal L belongs to an answer set $S$of Π, then there is some rule r in Π such that L belongs to the head of r and the body of r is satisfied by S.