# Lógica y Metodos avanzados de Razonamiento - Exercises 1

27 de Noviembre 2006

1. Given the following set of definite clauses:

   $$\begin{array}{l} slowsort(x, y) \leftarrow perm(x, y), sorted(y) \\ sorted(nil) \leftarrow \\ sorted(l(x, nil)) \leftarrow \\ sorted(l(x, l(y, z))) \leftarrow leq(x, y), sorted(l(y, z)) \\ perm(nil, nil) \leftarrow \\ perm(l(x, y), l(u, v)) \leftarrow delete(u, l(x, y), z), perm(z, v) \\ delete(x, l(x, y), y) \leftarrow \\ delete(x, l(y, z), l(y, w)) \leftarrow delete(x, z, w) \\ leq(0, x) \leftarrow \\ leq(s(x), s(y)) \leftarrow leq(x, y) \end{array}$$

   (a) Write down the clauses in First Order notation (Remark: $u$,$v$,$w$,$x$,$y$,$z$ are variables, $nil, 0$ are constants, $l, s$ are function symbols).

   (b) Find a model for this formula, i.e. specify an interpretation $\mathcal{I}$ which evaluates the formula to *true*

2. Transform the following formulae to clausal form using the three steps from the lecture and write down the resulting form as a set of clauses:

   - $\forall x \; \neg\exists y \; \forall z \; (P(h(z)) \rightarrow \neg Q(x, y))$
   - $\neg\forall x \; \exists y \; (Q(x, f(y)) \wedge Q(x, d) \rightarrow \neg R(x, g(y, f(z))))$
   - $\forall(Q(x) \vee ((R(a, x) \wedge P(y)) \vee (P(b) \wedge Q(b))))$

   *Remark: From the last example we see, that a naive transformation from DNF(disjunctive normal form) to CNF (conjunctive normal form) can cause significant (exponential in the worst case) blowup of the formula size.*

3. Construct the clausal form of the negated sentence $A \equiv \neg F$, where

   $$F \equiv (\forall x P(a, b, x) \wedge \forall u \forall v \exists w (P(u, v, w) \rightarrow R(w)) \wedge \forall y (R(y) \rightarrow R(f(y)))) \rightarrow \exists z R(f(z))$$

   Based on the clausal representation, prove $F$ by refuting $A$. Use the rules (S) and (R) from the lecture.

4. Find an mgu for each of the following pairs

   - $(\; p(u, v), \; p(y, f(y)) \;)$
   - $(\; p(a, x, f(g(y))), \; p(z, h(z, w), f(w)) \;)$
   - $(\; q(f(x, g(y, x)), a, g(b, z)), \; q(f(v, w), u, g(b, a)) \;)$

5. Try to unify the pair of atoms

   $$(P(u, u), P(f(x), x))$$

   using the unification algorithm from slide 35. What happens if you drop the occur-check?

6. (*) Prolog-Systems usually omit the expensive occur-check in the Unification algorithm. The following example shows why the occur check is so expensive. Try to unify the pair of atoms

$$(P(x_1, x_2, \dots, x_n), P(f(x_0, x_0), f(x_1, x_1), \dots, f(x_{n-1}, x_{n-1}))$$

What can you say about the occur check in the $k$-th iteration of the unification algorithm? *Remark: There are more efficient unification algorithms!*

7. (*) Generalize the Unification algorithm from the lecture slides to sets of clauses.

**Note that solving these exercises is for your benefit! You can send solutions and questions to me by Monday via e-mail:** ***axel.polleres@deri.org***