Lógica y Metodos avanzados de Razonamiento - Exercises 2

1 de noviembre 2006

In the following, we assume: $\exists, \forall$ bind stronger than $\wedge, \vee$; $\wedge, \vee$ bind stronger than $\rightarrow$.

I) Propositional Natural Deduction: Find proofs in natural deduction for the following sequents:

1. $(p \wedge q) \wedge r, s \wedge t \vdash q \wedge s$
2. $q \rightarrow (p \rightarrow r), \neg r, q \vdash \neg p$
3. $(p \rightarrow r) \wedge (q \rightarrow r) \vdash p \wedge q \rightarrow r$
4. $q \rightarrow r \vdash (p \rightarrow q) \rightarrow (p \rightarrow r)$
5. $p \rightarrow (q \rightarrow r), p \rightarrow q \vdash (p \rightarrow r)$
6. $p \rightarrow q, r \rightarrow s \vdash p \vee r \rightarrow q \wedge s$
7. $p \vee q \vdash r \rightarrow ((p \vee q) \wedge r)$
8. $(p \vee (q \rightarrow p)) \wedge q \vdash p$
9. $p \rightarrow q, r \rightarrow s \vdash p \wedge r \rightarrow q \wedge s$
10. $p \rightarrow q \vdash (p \wedge q \rightarrow p) \wedge p \rightarrow (p \wedge q)$
11. $\vdash (p \rightarrow q) \rightarrow ((r \rightarrow s) \rightarrow (p \wedge r \rightarrow q \wedge s))$
12. $(p \wedge q) \vee (p \wedge r) \vdash p \wedge (q \vee r)$
13. $\neg p \rightarrow \neg q \vdash q \rightarrow p$
14. $\neg p \vee \neg q \vdash \neg(p \wedge q)$
15. $\neg q, \neg r, p \rightarrow (q \vee r) \vdash \neg p$
16. $p \wedge \neg p \vdash \neg(r \rightarrow q) \wedge (r \rightarrow q)$
17. $\neg(\neg p \vee q) \vdash p$
18. $\vdash (p \rightarrow q) \vee (q \rightarrow r)$
19. $(s \rightarrow p) \vee (t \rightarrow q) \vdash (s \rightarrow q) \vee (t \rightarrow p)$

I) First-Order Natural Deduction: Find proofs in natural deduction for the following sequents:

1. $t_1 = t_2 \vdash t + t_2 = t + t_1$
2. $(x = 0) \vee (x + x > 0) \vdash (y = x + x) \rightarrow (y > 0) \vee (y = 0 + x)$
3. Let ϕ be a closed formula, prove:
   $\phi \rightarrow \forall x\ Q(x) \vdash \forall x\ (\phi \rightarrow Q(x))$
   Does this result still hold if x occurs freely in ϕ?
4. $\exists x\ (P(x) \wedge Q(x)) \vdash \exists x\ P(x) \wedge \exists x\ Q(x)$

Note that solving these exercises is for your benefit! You can send solutions and questions to me by Monday via e-mail: *axel@polleres.net*