

Unit 3 – Beyond Simple Entailment: Semantics of the RDF(S), Datatypes, and OWL vocabulary

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Unit Outline

1. RDF(S) Entailment: Giving semantics to the rdf: and rdfs: Vocabulary
2. D-Entailment: Giving Semantics to Datatypes
3. OWL Entailment: Giving Semantics to the owl: Vocabulary

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In the following, we use the following namespace prefixes:

```
@prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#> .  
@prefix rdfs: <http://www.w3.org/2000/01/rdf-schema#> .  
@prefix xsd: <http://www.w3.org/2001/XMLSchema#> .  
@prefix owl: <http://www.w3.org/2002/07/owl#> .
```

The `rdf:` vocabulary

The **RDF vocabulary**, `rdfV`, is a set of URI references in the `rdf:` namespace:

- `rdf:type` ... expresses class membership
- `rdf:Property` ... the class of properties
- `rdf:XMLLiteral` ... the datatype of XML Literals.
- `rdf:first`, `rdf:rest`, `rdf:nil`, `rdf:List` ... vocabulary to write lists (called “collections”) in RDF
- `rdf:Seq`, `rdf:Bag`, `rdf:Alt`, `rdf:_1`, `rdf:_2` ... vocabulary to describe (ordered, unordered, alternate) containers of values.
- `rdf:Statement`, `rdf:subject`, `rdf:predicate`, `rdf:object` ... vocabulary to make statements about statement (aka “reification”)
- `rdf:value` ... to describe structured values

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- `rdf:value` ... to describe structured values

Note: The first three are the most important for the formal semantics. The semantics doesn't make any strong restrictions on the grayed out vocabulary.

Collection vocabulary – Example

Collection vocabulary intended to be used to write lists, e.g.

```
ex:article ex:hasAuthors ( ex:johnny ex:jim ex:jack )
```

Collection vocabulary – Example

Collection vocabulary intended to be used to write lists, e.g.

```
ex:article ex:hasAuthors _:b1.  
_:b1 rdf:first ex:johnny ; rdf:rest _:b2.  
_:b2 rdf:first ex:jim ; rdf:rest _:b3.  
_:b3 rdf:first ex:jack ; rdf:rest rdf:nil.
```

Comes usually from RDF/XML's `parseType = "Collection"`.

Collection vocabulary – Example

Collection vocabulary intended to be used to write lists, e.g.

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ex:article ex:hasAuthors _:b1.  
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```

Comes usually from RDF/XML's `parseType = "Collection"`.

However, the semantics does not impose strong formal restrictions/conditions on the use of this vocabulary, e.g. a graph

```
ex:a rdf:first ex:a.
```

is fine (no restrictions on cyclic, incomplete, etc. lists.)

Container vocabulary – Example

Container vocabulary intended to be used to write containers of values, e.g.

```
ex:semWebCourse ex:hasParticipants [ a rdf:Bag;
                                     rdf:_1 ex:johny,
                                     rdf:_2 ex:jim;
                                     rdf:_3 ex:jack ] .

ex:semWebCourse ex:hasLecturer [ a rdf:Alt;
                                  rdf:_1 ex:prof1
                                  rdf:_2 ex:prof2 ] .
```

Comes usually from RDF/XML's `...`.

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ex:semWebCourse ex:hasParticipants [ a rdf:Bag;  
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```
ex:semWebCourse ex:hasLecturer [ a rdf:Alt;  
                                  rdf:_1 ex:prof1  
                                  rdf:_2 ex:prof2 ] .
```

Comes usually from RDF/XML's `...`.

However, the semantics does not impose strong formal restrictions/conditions on the use of this vocabulary, e.g. a graph.

```
ex:a rdf:_4 rdf:Seq;  
     rdf:_3 rdf:_3 .
```

is fine (e.g. no restrictions on multiple occurrences of a numbered membership property, etc.).

Reification vocabulary & rdf:value – Example

Reification vocab intended to be used to write statements about statements, e.g.

```
ex:axel ex:thinks [ rdf:subject dbpedia:Web_Ontology_Language;  
                  rdf:predicate owl:differentFrom  
                  rdf:object dbpedia:Description_Logics] .
```

Again, no semantics restrictions on the use of this vocabulary.

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Again, no semantics restrictions on the use of this vocabulary.

rdf:value intended to be used to denote “structured values”

```
ex:telephone1 ex:hasPrice [ rdf:value 20; ex:currency ‘EUR’]
```

Again, no semantics restrictions on the use of this vocabulary.

Reification vocabulary & rdf:value – Example

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Purely speculative statement by the lecturer

Intuitively, all these parts of the vocabulary have been left without fixed semantics for the moment... may be changed/replaced by later standards? e.g. “Reification-Entailment”, etc. cf. also <http://www.w3.org/2011/rdf-wg/track/issues/25>

... now to the formal part.

Recall from last time (lecture 2, slide 24):

Simple Interpretation

A **simple interpretation** I over vocabulary V is a 6-tuple
 $I = \langle IR, IP, IEXT, IS, IL, LV \rangle$

rdf-interpretation

An **rdf-interpretation** is a simple interpretation

$I = \langle IR, IP, IEXT, IS, IL, LV \rangle$ which fulfills the following conditions and all the following set of **axiomatic triples** A_{rdf} :

RDF semantic conditions.

x is in IP if and only if $\langle x, I(\text{rdf:Property}) \rangle$ is in $IEXT(I(\text{rdf:type}))$

If $"xxx" \wedge \text{rdf:XMLLiteral}$ is in V and xxx is a well-typed XML literal string, then

$IL("xxx" \wedge \text{rdf:XMLLiteral})$ is the XML value of xxx ;
 $IL("xxx" \wedge \text{rdf:XMLLiteral})$ is in LV ;
 $IEXT(I(\text{rdf:type}))$ contains $\langle IL("xxx" \wedge \text{rdf:XMLLiteral}), I(\text{rdf:XMLLiteral}) \rangle$

If $"xxx" \wedge \text{rdf:XMLLiteral}$ is in V and xxx is an ill-typed XML literal string, then

$IL("xxx" \wedge \text{rdf:XMLLiteral})$ is not in LV ;
 $IEXT(I(\text{rdf:type}))$ does not contain $\langle IL("xxx" \wedge \text{rdf:XMLLiteral}), I(\text{rdf:XMLLiteral}) \rangle$

RDF axiomatic triples.

```

rdf:type rdf:type rdf:Property .
rdf:subject rdf:type rdf:Property .
rdf:predicate rdf:type rdf:Property .
rdf:object rdf:type rdf:Property .
rdf:first rdf:type rdf:Property .
rdf:rest rdf:type rdf:Property .
rdf:value rdf:type rdf:Property .
rdf:_1 rdf:type rdf:Property .
rdf:_2 rdf:type rdf:Property .
...
rdf:nil rdf:type rdf:List .

```


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rdf:rest rdf:type rdf:Property .
rdf:value rdf:type rdf:Property .
rdf:_1 rdf:type rdf:Property .
rdf:_2 rdf:type rdf:Property .
...
rdf:nil rdf:type rdf:List .

```

rdf-satisfaction, entailment

written \models_{rdf} defined analogously to \models , but with respect to rdf-interpretations.

rdf-interpretation

Some consequences:

Proposition 1

If $s \ p \ o. \in G$ then $G \models_{rdf} \{p \text{ a } rdf:Property\}$.

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Particularly, note that the first axiomatic triple is redundant!

Let L_{XML} denote the set of well-formed XML literals.

Proposition 2

If $s \text{ p } "xyz" \wedge rdf:XMLLiteral. \in G$, such that $xyz \in L_{XML}$ then $G \models_{rdf} \{[] \text{ a } rdf:XMLLiteral\}$.

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Remark: ill-formed XML literals are hardly (impossibly?) writeable in RDF/XML, but in other syntaxes.

Computing entailment for \models_{rdf}

Recall main result for simple entailment (\models):

Interpolation Lemma

$S \models E$ if and only if a subgraph of S is an instance of E .

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Does no longer hold for \models_{rdf} :-)

BUT! Can be enabled by rules!

rdf-closure ($Cl_{rdf}(G)$)

The **rdf-closure** $Cl_{rdf}(G)$ of graph G is defined by $G \uplus A_{rdf}$ plus exhaustive application of the following rules to G :

- If $s \text{ p } o. \in Cl_{rdf}(G)$ then add **p a rdf:Property**.
- If $s \text{ p } "xyz" \wedge \text{rdf:XMLElement} \in Cl_{rdf}(G)$ and $xyz \in L_{XML}$ then add **__ :xyz a rdf:XMLElement**, where **__ :xyz** is a distinct bnode for that literal.

Slight abstraction of derivation rules *rdf1* and *rdf2* from the spec.

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- p a `rdf:Property` . $\leftarrow s p o$.
- `_:xyz` a `rdf:XMLLiteral` . $\leftarrow s p \text{"xyz"}^{\wedge} \text{rdf:XMLLiteral}$.

Let's write them in more "rule" syntax.

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- $\forall s, p, o (triple(p, rdf:type, rdf:Property) \leftarrow triple(s, p, o))$
- $\forall s, p \exists b (triple(b, rdf:type, rdf:XMLLiteral) \leftarrow triple(s, p, "xyz" \wedge rdf:XMLLiteral))$

Or as first-order sentences.

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- `triple(P,rdf:type,rdf:Property) :- triple(S,P,0).`
- `triple(bxyz,rdf:type,rdf:XMLLiteral) :- triple(S,P,"xyz"^^rdf:XMLLiteral)`

Or as Datalog rules.

rdf-entailment: Properties

RDF entailment lemma (slightly adapted from the spec)

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But: A finite subset of $Cl_{rdf}(G)$ is sufficient! Let the **reduced closure of G_1 w.r.t. G_2** , written $Cl_{rdf}(G_1, G_2)$, be defined as $Cl_{rdf}(G_1)$ with only those axiomatic triples $rdf:_n \quad rdf:type \quad rdf:Property$. where $rdf:_n \in V(G_2)$

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\Rightarrow For a finite graph G_2 , $Cl_{rdf}(G_1, G_2)$ is always finite.

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Modified RDF entailment lemma (slightly adapted from the spec)

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Modified RDF entailment lemma (slightly adapted from the spec)

$S \models_{rdf} E$ if and only if $Cl_{rdf}(S, E) \models E$

Means that we can compute RDF entailment finitely! :-)

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... Didn't buy us too much, except that:

- entailment became more complicated
- we have a way to express that a predicate is an `rdf:Property`
- we have a way to express that XML Literals are `rdf:XMLLiterals`

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- we have a way to express that XML Literals are `rdf:XMLLiterals`

Still, we cannot **describe other vocabularies** \Rightarrow **`rdfs:`**

The rdfs: vocabulary

The **RDFS vocabulary**, `rdfsV`, extends `rdfV`:

- `rdfs:domain`, `rdfs:range`
- `rdfs:subClassOf`, `rdfs:subPropertyOf`
- `rdfs:Resource` ... all resources.
- `rdfs:Class` ... all resources which denote classes.
- `rdfs:Literal` ... the “class of literal values”.
- `rdfs:Datatype` ... the “class of datatypes”.
- `rdfs:Container` ... a joint superclass of `rdf:Alt`, `rdf:Seq`, `rdf:Bag`.
- `rdfs:ContainerMembershipProperty` ... the “class of container membership predicates” (`rdf:_1`, `rdf:_2`, ...).
- `rdfs:member` a joint superproperty of `rdf:_1`, `rdf:_2`, ...
- `rdfs:comment`, `rdfs:seeAlso`, `rdfs:isDefinedBy`, `rdfs:label` some more conventional vocabulary without strong “formal” semantics¹

¹for illustrative examples, let's check the FOAF ontology

RDF/XML version: <http://xmlns.com/foaf/spec/20071002.rdf>

Extracted Turtle version:

http://www.polleres.net/teaching/SemWebTech_2009/testdata/foafOntology.ttl

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Additionally to IR and IP , the semantic conditions for RDF also talk about the **set of classes**, denoted by $IC \subseteq IR$:

IC

IC is defined as the set of resources which appear in the range of the extension of `rdf:type`, i.e. all the y such that $\langle x, y \rangle \in IEXT(I(\text{rdf:type}))$.

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For brevity, we also define:

$ICEXT$

$ICEXT : IC \rightarrow 2^{IR}$ defined as $x \in ICEXT(y) \Leftrightarrow \langle x, y \rangle \in IEXT(I(\text{rdf:type}))$

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For brevity, we also define:

$ICEXT$

$ICEXT : IC \rightarrow 2^{IR}$ defined as $x \in ICEXT(y) \Leftrightarrow \langle x, y \rangle \in IEXT(I(\text{rdf:type}))$

Let's look into the **semantic conditions** and **axiomatic triples** for rdfs-interpretations now...

rdfs-interpretations: Semantic conditions

x is in $\text{ICEXT}(y)$ if and only if $\langle x,y \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:type}))$

$\text{IC} = \text{ICEXT}(\text{I}(\text{rdfs:Class}))$

$\text{IR} = \text{ICEXT}(\text{I}(\text{rdfs:Resource}))$

$\text{LV} = \text{ICEXT}(\text{I}(\text{rdfs:Literal}))$

If $\langle x,y \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:domain}))$ and $\langle u,v \rangle$ is in $\text{IEXT}(x)$ then u is in $\text{ICEXT}(y)$

If $\langle x,y \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:range}))$ and $\langle u,v \rangle$ is in $\text{IEXT}(x)$ then v is in $\text{ICEXT}(y)$

$\text{IEXT}(\text{I}(\text{rdfs:subPropertyOf}))$ is transitive and reflexive on IP

If $\langle x,y \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:subPropertyOf}))$ then x and y are in IP and $\text{IEXT}(x)$ is a subset of $\text{IEXT}(y)$

If x is in IC then $\langle x, \text{I}(\text{rdfs:Resource}) \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:subClassOf}))$

If $\langle x,y \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:subClassOf}))$ then x and y are in IC and $\text{ICEXT}(x)$ is a subset of $\text{ICEXT}(y)$

$\text{IEXT}(\text{I}(\text{rdfs:subClassOf}))$ is transitive and reflexive on IC

If x is in $\text{ICEXT}(\text{I}(\text{rdfs:ContainerMembershipProperty}))$ then:
 $\langle x, \text{I}(\text{rdfs:member}) \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:subPropertyOf}))$

If x is in $\text{ICEXT}(\text{I}(\text{rdfs:Datatype}))$ then $\langle x, \text{I}(\text{rdfs:Literal}) \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:subClassOf}))$

rdfs-interpretations: Semantic conditions

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If $\langle x,y \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:subClassOf}))$ then x and y are in IC and $\text{ICEXT}(x)$ is a subset of $\text{ICEXT}(y)$

$\text{IEXT}(\text{I}(\text{rdfs:subClassOf}))$ is transitive and reflexive on IC

If x is in $\text{ICEXT}(\text{I}(\text{rdfs:ContainerMembershipProperty}))$ then:
 $\langle x, \text{I}(\text{rdfs:member}) \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:subPropertyOf}))$

If x is in $\text{ICEXT}(\text{I}(\text{rdfs:Datatype}))$ then $\langle x, \text{I}(\text{rdfs:Literal}) \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:subClassOf}))$

o a Class . $\Leftarrow s$ a o .

$[]$ a s . $\Leftarrow s$ a Class .

s a Resource . $\Leftarrow s$ p o .

p a Resource . $\Leftarrow s$ p o .

o a Resource . $\Leftarrow s$ p o .

$_ : 1$ a Literal . $\Leftarrow s$ p l .

s p $_ : 1$. $\Leftarrow s$ p l .

$o \notin L$
 $l \in L$
 $l \in L$

rdfs-interpretations: Semantic conditions

x is in $\text{ICEXT}(y)$ if and only if $\langle x, y \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:type}))$

$\text{IC} = \text{ICEXT}(\text{I}(\text{rdfs:Class}))$

$\text{IR} = \text{ICEXT}(\text{I}(\text{rdfs:Resource}))$

$\text{LV} = \text{ICEXT}(\text{I}(\text{rdfs:Literal}))$

If $\langle x, y \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:domain}))$ and $\langle u, v \rangle$ is in $\text{IEXT}(x)$ then u is in $\text{ICEXT}(y)$

If $\langle x, y \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:range}))$ and $\langle u, v \rangle$ is in $\text{IEXT}(x)$ then v is in $\text{ICEXT}(y)$

$\text{IEXT}(\text{I}(\text{rdfs:subPropertyOf}))$ is transitive and reflexive on IP

If $\langle x, y \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:subPropertyOf}))$ then x and y are in IP and $\text{IEXT}(x)$ is a subset of $\text{IEXT}(y)$

If x is in IC then $\langle x, \text{I}(\text{rdfs:Resource}) \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:subClassOf}))$

If $\langle x, y \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:subClassOf}))$ then x and y are in IC and $\text{ICEXT}(x)$ is a subset of $\text{ICEXT}(y)$

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 $\langle x, \text{I}(\text{rdfs:member}) \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:subPropertyOf}))$

If x is in $\text{ICEXT}(\text{I}(\text{rdfs:Datatype}))$ then $\langle x, \text{I}(\text{rdfs:Literal}) \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:subClassOf}))$

o a Class . $\Leftarrow s$ a o .

$[\]$ a s . $\Leftarrow s$ a Class .

s a Resource . $\Leftarrow s$ p o .

p a Resource . $\Leftarrow s$ p o .

o a Resource . $\Leftarrow s$ p o .

$_ : 1$ a Literal . $\Leftarrow s$ p l .

s p $_ : 1$. $\Leftarrow s$ p l .

$o \notin L$
 $l \in L$
 $l \in L$

s a c . $\Leftarrow p$ domain c . s p o .

rdfs-interpretations: Semantic conditions

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$\text{LV} = \text{ICEXT}(\text{I}(\text{rdfs:Literal}))$

o a Class . $\Leftarrow s$ a o .

$[\]$ a s . $\Leftarrow s$ a Class .

s a Resource . $\Leftarrow s$ p o .

p a Resource . $\Leftarrow s$ p o .

o a Resource . $\Leftarrow s$ p o .

$_ : l$ a Literal . $\Leftarrow s$ p l .

s p $_ : l$. $\Leftarrow s$ p l .

$o \notin L$

$l \in L$

$l \in L$

If $\langle x,y \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:domain}))$ and $\langle u,v \rangle$ is in $\text{IEXT}(x)$ then u is in $\text{ICEXT}(y)$

s a c . $\Leftarrow p$ domain c . s p o .

If $\langle x,y \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:range}))$ and $\langle u,v \rangle$ is in $\text{IEXT}(x)$ then v is in $\text{ICEXT}(y)$

o a c . $\Leftarrow p$ range c . s p o . $o \notin L$

$\text{IEXT}(\text{I}(\text{rdfs:subPropertyOf}))$ is transitive and reflexive on IP

If $\langle x,y \rangle$ is in $\text{IEXT}(\text{I}(\text{rdfs:subPropertyOf}))$ then x and y are in IP and $\text{IEXT}(x)$ is a subset of $\text{IEXT}(y)$

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rdfs-interpretations: Semantic conditions

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$o \notin L$

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s a c . $\Leftarrow p$ domain c . s p o .

o a c . $\Leftarrow p$ range c . s p o . $o \notin L$

s subPropertyOf r . $\Leftarrow s$ subPropertyOf o .
 o subPropertyOf r .

s subPropertyOf s . $\Leftarrow s$ a Property .

s a Property . $\Leftarrow s$ subPropertyOf o .

o a Property . $\Leftarrow s$ subPropertyOf o .

s q $o \Leftarrow p$ subPropertyOf q . s p o . $q \notin BL$

s subClassOf Resource . $\Leftarrow s$ a Class .

s a Class . $\Leftarrow s$ subClassOf o .

o a Class . $\Leftarrow s$ subClassOf o . $o \notin L$

s a c . $\Leftarrow o$ suClassOf c . s a o .

s subClassOf r . $\Leftarrow s$ subClassOf o .

o subClassOf r .

s subClassOf s . $\Leftarrow s$ a Property .

s subPropertyOf member . \Leftarrow

s a ContainerMembershipProperty .

s subClassOf Literal . $\Leftarrow s$ a Datatype .

rdfs-interpretations: Additional Axiomatic Triples

 A_{rdfs}

<code>rdf:type</code>	<code>rdfs:domain rdfs:Resource;</code> <code>rdfs:range rdfs:Class .</code>		
<code>rdfs:domain</code>	<code>rdfs:domain rdf:Property;</code> <code>rdfs:range rdfs:Class .</code>	<code>rdf:subject</code>	<code>rdfs:domain rdf:Statement;</code> <code>rdfs:range rdfs:Resource .</code>
<code>rdfs:range</code>	<code>rdfs:domain rdf:Property;</code> <code>rdfs:range rdfs:Class .</code>	<code>rdf:predicate</code>	<code>rdfs:domain rdf:Statement;</code> <code>rdfs:range rdfs:Resource .</code>
<code>rdfs:subPropertyOf</code>	<code>rdfs:domain rdf:Property;</code> <code>rdfs:range rdf:Property .</code>	<code>rdf:object</code>	<code>rdfs:domain rdf:Statement;</code> <code>rdfs:range rdfs:Resource .</code>
<code>rdfs:subClassOf</code>	<code>rdfs:domain rdfs:Class;</code> <code>rdfs:range rdfs:Class .</code>		<code>rdf:Alt rdfs:subClassOf rdfs:Container .</code> <code>rdf:Bag rdfs:subClassOf rdfs:Container .</code> <code>rdf:Seq rdfs:subClassOf rdfs:Container .</code> <code>rdfs:ContainerMembershipProperty</code> <code> rdfs:subClassOf rdf:Property .</code>
<code>rdfs:member</code>	<code>rdfs:domain rdfs:Resource;</code> <code>rdfs:range rdfs:Resource .</code>		<code>rdfs:isDefinedBy rdfs:subPropertyOf rdfs:seeAlso .</code>
<code>rdfs:seeAlso</code>	<code>rdfs:domain rdfs:Resource;</code> <code>rdfs:range rdfs:Resource .</code>		<code>rdf:XMLLiteral rdf:type rdfs:Datatype .</code> <code>rdf:XMLLiteral rdfs:subClassOf rdfs:Literal .</code> <code>rdfs:Datatype rdfs:subClassOf rdfs:Class .</code>
<code>rdfs:isDefinedBy</code>	<code>rdfs:domain rdfs:Resource;</code> <code>rdfs:range rdfs:Resource .</code>		<code>rdf:_1 rdf:type rdfs:ContainerMembershipProperty .</code> <code>rdf:_1 rdfs:domain rdfs:Resource .</code> <code>rdf:_1 rdfs:range rdfs:Resource .</code> <code>rdf:_2 rdf:type rdfs:ContainerMembershipProperty .</code> <code>rdf:_2 rdfs:domain rdfs:Resource .</code> <code>rdf:_2 rdfs:range rdfs:Resource .</code> <code>...</code>
<code>rdf:value</code>	<code>rdfs:domain rdfs:Resource;</code> <code>rdfs:range rdfs:Resource .</code>		
<code>rdfs:comment</code>	<code>rdfs:domain rdfs:Resource;</code> <code>rdfs:range rdfs:Literal .</code>		
<code>rdfs:label</code>	<code>rdfs:domain rdfs:Resource;</code> <code>rdfs:range rdfs:Literal .</code>		
<code>rdf:first</code>	<code>rdfs:domain rdf:List;</code> <code>rdfs:range rdfs:Resource .</code>		
<code>rdf:rest</code>	<code>rdfs:domain rdf:List .</code> <code>rdfs:range rdf:List .</code>		

rdfs-interpretations: Axiomatic Triples

A_{rdfs} has some “somewhat redundant triples” (let’s call the “gist” A_{rdfs}^-)

		<code>rdf:subject</code>	<code>rdfs:domain rdf:Statement;</code>
		<code>rdf:predicate</code>	<code>rdfs:domain rdf:Statement;</code>
		<code>rdf:object</code>	<code>rdfs:domain rdf:Statement;</code>
<code>rdf:type</code>	<code>rdfs:range rdfs:Class .</code>		
<code>rdfs:domain</code>	<code>rdfs:range rdfs:Class .</code>		
<code>rdfs:range</code>	<code>rdfs:range rdfs:Class .</code>		
<code>rdfs:subClassOf</code>	<code>rdfs:domain rdfs:Class;</code> <code>rdfs:range rdfs:Class .</code>	<code>rdf:Alt rdfs:subClassOf rdfs:Container .</code> <code>rdf:Bag rdfs:subClassOf rdfs:Container .</code> <code>rdf:Seq rdfs:subClassOf rdfs:Container .</code> <code>rdfs:ContainerMembershipProperty</code> <code> rdfs:subClassOf rdf:Property .</code>	
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Actually, one could think: A_{rdfs}^- is enough:

- Axiomatic triples s `domain/range rdfs:Resource .`
- Axiomatic triples p `domain/range rdf:Property .`

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		<code>rdf:Bag</code>	<code>rdfs:subClassOf rdfs:Container .</code>
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		<code>rdfs:ContainerMembershipProperty</code>	<code>rdfs:subClassOf rdf:Property .</code>
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<code>rdf:rest</code>	<code>rdfs:domain rdf:List .</code> <code>rdfs:range rdf:List .</code>	<code>rdf:_1</code>	<code>rdf:type rdfs:ContainerMembershipProperty .</code>
		<code>rdf:_2</code>	<code>rdf:type rdfs:ContainerMembershipProperty .</code>
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More on that later. **bottomline: if we aren't interested in inferring triples**
 x `rdfs:domain rdfs:Resource.` , we can probably do without these.

rdfs-entailment: Properties

Recall:

Modified RDF entailment lemma (slightly adapted from the spec)

$S \models_{rdfs} E$ if and only if $Cl_{rdfs}(S, E) \models E$

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... but, means that we can compute RDFS entailment finitely! :-)

Spec entailment rules are – incomplete!

Some of the rules we had before had restrictions in the antecedent, e.g.

$s \text{ a } c . \Leftarrow p \text{ domain } c . \quad s \text{ p } o .$
 $s \text{ q } o \Leftarrow p \text{ subPropertyOf } q . \quad s \text{ p } o .$

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Problem: If we execute the rules with the restrictions in place, we won't get:

```
s rdf:type c.
```

The invalid “intermediate” triple

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$$_ : p \text{ rdfs:domain } c .$$

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The invalid “intermediate” triple

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Solution: easy, (i) allow “generalized” RDF ($UBL \times UBL \times UBL$) for intermediate computation of closure, or (ii) use modified “implicit typing rule”, cf. [Muñoz *et al.*, 2007]:

$$s \text{ a } c \Leftarrow p \text{ subPropertyOf } q . \quad q \text{ rdfs:domain } c . \quad s \text{ p } o .$$

Inconsistency

Proposition 3

ANY Graph

- containing a triple $s \ p \ "xyz" \wedge \text{rdf:XMLLiteral}$. such that xyz is NOT a well-formed XML literal and
- allowing to infer the triple $p \ \text{rdf:range} \ \text{rdf:XMLLiteral}$.

is **false in any rdfs-interpretation.**

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Why?

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Why?

- by semantic conditions of rdf- and rdfs-interpretations!

Implication:

- There can be rdfs-inconsistent RDF graphs, e.g.
 - ex:a ex:p "<notLegalXML"^^rdf:XMLLiteral .
 - ex:p rdfs:range rdf:XMLLiteral .

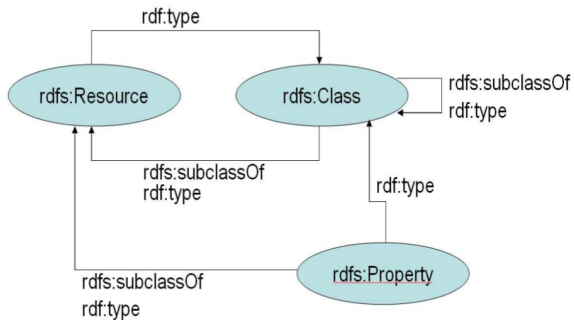
rdfs-entailment: Example 1

$\{\} \models_{rdfs} G_5$

G_5 :

```
rdfs:Resource rdf:type rdfs:Class.  
rdf:Property rdf:type rdfs:Resource.  
rdf:Property rdf:subClassOf rdfs:Resource.  
rdf:Property rdf:type rdfs:Class.  
rdfs:Class rdf:type rdfs:Resource.  
rdfs:Class rdf:type rdfs:Class.  
rdfs:Class rdf:subClassOf rdfs:Resource.  
rdfs:Class rdf:subClassOf rdfs:Class.
```

rdfs-entailment: Example 1

 $\{\} \models_{rdfs} G_5$ $G_5 :$ 

rdfs-entailment: Example 2

$$G_1 \models_{rdfs} G_2$$
 $G_1 :$

```
ex:alice foaf:knows ex:bob.  
ex:alice foaf:name "Alice".  
foaf:knows rdfs:domain foaf:Person.
```

 $G_2 :$

```
ex:alice rdf:type foaf:Person.
```

rdfs-entailment: Example 3

$$\{ \} \models_{rdfs} \{ xxx \text{ rdf:type rdfs:Resource. } \}$$

for all xxx in the Vocabulary of any interpretation I .

Implication: Since all rdfs-Interpretations are over an infinite vocabulary (rdf:_n), even without the axiomatic triples that would already entail infinite triples.

Unit Outline

1. RDF(S) Entailment: Giving semantics to the rdf: and rdfs: Vocabulary
2. D-Entailment: Giving Semantics to Datatypes
3. OWL Entailment: Giving Semantics to the owl: Vocabulary

Datatype

A **datatype** d is an entity characterized by a set of character strings called lexical forms (or *lexical space*) and a mapping from that set to a set of values (called *value space*). Exactly how these sets and mappings (called **lexical-to-value mapping** $L2V(d)$) are defined is a matter external to RDF, e.g. XML Schema [XML Schema Datatypes, 2001].

Datatype

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A *datatype map* is a set of datatypes.

Besides `rdf:XMLLiteral`, the RDF spec suggests to support the following XML Schema datatypes:

XSD datatypes

[xsd:string](#), [xsd:boolean](#), [xsd:decimal](#), [xsd:float](#), [xsd:double](#), [xsd:dateTime](#), [xsd:time](#), [xsd:date](#), [xsd:gYearMonth](#), [xsd:gYear](#), [xsd:gMonthDay](#), [xsd:gDay](#), [xsd:gMonth](#), [xsd:hexBinary](#), [xsd:base64Binary](#), [xsd:anyURI](#), [xsd:normalizedString](#), [xsd:token](#), [xsd:language](#), [xsd:NMTOKEN](#), [xsd:Name](#), [xsd:NCName](#), [xsd:integer](#), [xsd:nonPositiveInteger](#), [xsd:negativeInteger](#), [xsd:long](#), [xsd:int](#), [xsd:short](#), [xsd:byte](#), [xsd:nonNegativeInteger](#), [xsd:unsignedLong](#), [xsd:unsignedInt](#), [xsd:unsignedShort](#), [xsd:unsignedByte](#), [xsd:positiveInteger](#)

D-interpretations

D-interpretation

A **D-interpretation** is an rdfs-interpretation I defined by the following additional semantic conditions:

General semantic conditions for datatypes.

if $\langle \text{aaa}, x \rangle$ is in D then $I(\text{aaa}) = x$

if $\langle \text{aaa}, x \rangle$ is in D then $\text{ICEXT}(x)$ is the value space of x and is a subset of LV

if $\langle \text{aaa}, x \rangle$ is in D then for any typed literal "sss"^^ddd in V with $I(\text{ddd}) = x$,
if sss is in the lexical space of x then $\text{IL}(\text{"sss"^^ddd}) = \text{L2V}(x)(\text{sss})$, otherwise $\text{IL}(\text{"sss"^^ddd})$ is not in LV

if $\langle \text{aaa}, x \rangle$ is in D then $I(\text{aaa})$ is in $\text{ICEXT}(I(\text{rdfs:Datatype}))$

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if $\langle \text{aaa}, x \rangle$ is in D then $I(\text{aaa})$ is in $\text{ICEXT}(I(\text{rdfs:Datatype}))$

- In principle, just generalization of the Semantic conditions for `rdf:XMLLiteral` to other datatypes.
- Inuitive Reading: fix the interpretation of typed literals according to $L2V(d)$

That's it!

D-entailment (\models_D) – Examples
$$\{ s \ p \ "2.0" \wedge \text{xsd:decimal.} \} \models_D \{ s \ p \ "2" \wedge \text{xsd:integer.} \}$$
$$\{ s \ p \ "2" \wedge \text{xsd:integer.} \} \models_D \{ s \ p \ "2.0" \wedge \text{xsd:decimal.} \}$$

D-entailment (\models_D) – Examples

`{ s p "2.0"^^xsd:decimal. } \models_D { s p "2"^^xsd:integer. }`

`{ s p "2"^^xsd:integer. } \models_D { s p "2.0"^^xsd:decimal. }`

D-inconsistent graphs:

`{ ex:a ex:b "25"^^xsd:decimal . ex:b rdfs:range xsd:string .}`
 (because the value spaces of `sd:string` and `xsd:decimal` are disjoint)

`ex:a ex:p "2.5"^^xsd:decimal . ex:p rdfs:range xsd:integer .}`
 (overlapping value spaces, but 2.5 outside `xsd:integer`'s value space)

`{ ex:a ex:b "haha"^^xsd:decimal. }`

(outside lexical space, i.e. "ill-formed" for `xsd:decimal`)

Rule-based computation of D-entailment?

- Basically, needs an oracle to evaluate $L2V$...

e.g. could be done by a built-in that maps all typed literals in a graph to compute $L2V$ of their value:

$$s \text{ p } L2V(l) \text{ .} \Leftarrow s \text{ p } l^{\wedge} d \text{ .} \quad \text{for } l \text{ in the lex. space of } d.$$

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Again needs “generalized intermediate triples”

($UBL \cup V_s \times UBL \cup V_s \times UBL \cup V_s$) for intermediate computation of closure, where V_s is the union of the value spaces of all supported datatypes.

Unit Outline

1. RDF(S) Entailment: Giving semantics to the rdf: and rdfs: Vocabulary
2. D-Entailment: Giving Semantics to Datatypes
3. OWL Entailment: Giving Semantics to the owl: Vocabulary

Semantics of the owl:Vocabulary

Not part of the RDF Spec, separate document, for the moment, we will restrict ourselves to OWL Version 1 [Patel-Schneider *et al.*, 2004].

- OWL adds the possibility to add some DL axioms (TBox, ABox) to RDF ($SHOIN(\mathcal{D})$).
- OWL defines a rewriting how to write such DL axioms in RDF.
- OWL adds the possibility to add some DL statements to an ontology.
- Defines two semantics:
 - DL-style model-theoretic semantics, for the syntactic subset of OWL which corresponds to DL
 - RDF compatible semantics

OWL DL in two slides: 1/2

Expressing property characteristics:

OWL property axioms as RDF triples	DL syntax	FOL short representation
$P \text{ rdfs:domain } C .$	$\top \sqsubseteq \forall P^- . C$	$\forall x, y. P(x, y) \supset C(x)$
$P \text{ rdfs:range } C .$	$\top \sqsubseteq \forall P . C$	$\forall x, y. P(x, y) \supset C(y)$
$P \text{ owl:inverseOf } P_0 .$	$P \equiv P_0^-$	$\forall x, y. P(x, y) \equiv P_0(y, x)$
$P \text{ rdf:type owl:SymmetricProperty.}$	$P \equiv P^-$	$\forall x, y. P(x, y) \equiv P(y, x)$
$P \text{ rdf:type owl:FunctionalProperty.}$	$\top \sqsubseteq \leq 1P$	$\forall x, y, z. P(x, y) \wedge P(x, z) \supset y = z$
$P \text{ rdf:type owl:InverseFunctionalProperty.}$	$\top \sqsubseteq \leq 1P^-$	$\forall x, y, z. P(x, y) \wedge P(z, y) \supset x = z$
$P \text{ rdf:type owl:TransitiveProperty.}$	$P^+ \sqsubseteq P$	$\forall x, y, z. P(x, y) \wedge P(y, z) \supset P(x, z)$

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Expressing complex class descriptions:

OWL complex class descriptions*	DL syntax	FOL short representation
owl:Thing	\top	$x = x$
owl:Nothing	\perp	$\neg x = x$
owl:intersectionOf ($C_1 \dots C_n$)	$C_1 \sqcap \dots \sqcap C_n$	$C_1(x) \wedge \dots \wedge C_n(x)$
owl:unionOf ($C_1 \dots C_n$)	$C_1 \sqcup \dots \sqcup C_n$	$C_1(x) \vee \dots \vee C_n(x)$
owl:complementOf (C)	$\neg C$	$\neg C(x)$
owl:oneOf ($o_1 \dots o_n$)	$\{o_1, \dots, o_n\}$	$x = o_1 \vee \dots \vee x = o_n$
owl:restriction ($P \text{ owl:someValuesFrom } (C)$)	$\exists P . C$	$\exists y. P(x, y) \wedge C(y)$
owl:restriction ($P \text{ owl:allValuesFrom } (C)$)	$\forall P . C$	$\forall y. P(x, y) \supset C(y)$
owl:restriction ($P \text{ owl:value } (o)$)	$\exists P . \{o\}$	$P(x, o)$
owl:restriction ($P \text{ owl:minCardinality } (n)$)	$\geq nP$	$\exists y_1 \dots y_n . \bigwedge_{k=1}^n P(x, y_k) \wedge \bigwedge_{i < j} y_i \neq y_j$
owl:restriction ($P \text{ owl:maxCardinality } (n)$)	$\leq nP$	$\forall y_1 \dots y_{n+1} . \bigwedge_{k=1}^{n+1} P(x, y_k) \supset \bigvee_{i < j} y_i = y_j$

*For reasons of legibility, we use a variant of the OWL abstract syntax [Patel-Schneider et al., 2004] in this table.

OWL DL in two slides: 2/2

Relating Class descriptions:

 C_1 rdfs:subClassOf C_2 C_1 owl:equivalentClass C_2 C_1 owl:disjointWith C_2 $C_1 \sqsubseteq C_2$ $C_1 \equiv C_2$ $C_1 \sqcap C_2 \sqsubseteq \perp$

Relating individuals:

 o_1 owl:sameAs o_1 o_1 owl:differentFrom o_2 $o_1 = o_2$ $o_1 \neq o_2$

OWL DL in two slides: 2/2

Relating Class descriptions:

C_1 rdfs:subClassOf C_2	$C_1 \sqsubseteq C_2$
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Relating individuals:

o_1 owl:sameAs o_1	$o_1 = o_2$
o_1 owl:differentFrom o_2	$o_1 \neq o_2$

Let's look into some examples of

- How to write OWL in RDF.
- what it means
- possible problems

How to write OWL in RDF – A simple ontology about reviewers:

- **Properties:** title, isAuthorOf, publishedIn, etc.
- **Classes:** Senior, Paper, Publication, etc.
- **Relations:**
 - *A Publication is a Paper which has been published* (subclass + existential condition on property)
 - *isAuthorOf is the opposite of Dublin Core's dc:creator Property*²
 - *A Senior researcher is a foaf:Person who isAuthorOf 10+ Publications* (subclass + condition on cardinality)
 - *Each item can be publishedIn at most one venue* (functional property)
 -
 -

²reuse of external ontologies!

OWL Example: A simple ontology about reviewers: in DL

$$\exists ex:title.\top \sqsubseteq ex:Paper \quad (i)$$

$$\exists ex:title^-\top \sqsubseteq xsd:string \quad (ii)$$

$$ex:isAuthorOf^- \equiv dc:creator \quad (iii)$$

$$ex:Publication \equiv ex:Paper \sqcap \exists ex:publishedIn.\top \quad (iv)$$

$$\top \sqsubseteq \leq 1 \ ex:publishedIn^- \quad (v)$$

$$ex:Senior \equiv foaf:Person \sqcap \geq 10 \ ex:isAuthorOf \sqcap \quad (vi)$$

$$\exists ex:isAuthorOf.ex:Publication$$

$$ex:Club100 \equiv foaf:Person \sqcap \geq 100 \ ex:isAuthorOf \quad (vii)$$

OWL Example: A simple ontology about reviewers: in OWL/RDF

■ $\exists ex:title.\top \sqsubseteq ex:Paper$

(i)

OWL Example: A simple ontology about reviewers: in OWL/RDF

- $\exists ex:title.\top \sqsubseteq ex:Paper$

```
[a owl:Restriction; owl:onProperty ex:title; owl:someValuesFrom owl:Thing]  
rdfs:subclassOf ex:Paper .
```

(i)

OWL Example: A simple ontology about reviewers: in OWL/RDF

■ $\exists ex:title.\top \sqsubseteq ex:Paper$

(i)

`ex:title rdfs:domain dc:creator .`

OWL Example: A simple ontology about reviewers: in OWL/RDF

- $\exists ex:title.\top \sqsubseteq ex:Paper$ (i)
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`ex:isAuthorOf owl:inverseOf dc:creator .`

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`ex:Publication owl:intersectionOf (ex:Paper [a owl:Restriction; owl:onProperty ex:publishedIn ; owl:minCardinality 1]) .`

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`owl:Thing rdfs:subclassOf [a owl:restriction; on Property ex:publishedIn; owl:maxCardinality 1] .`

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`ex:publishedIn a owl:FunctionalProperty .`

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- $ex:Senior \equiv foaf:Person \sqcap \geq 10 ex:isAuthorOf \sqcap \exists ex:isAuthorOf.ex:Publication$ (vi)

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`ex:Senior owl:intersectionOf (foaf:Person [a owl:Restriction; owl:onProperty ex:isAuthorOf ; owl:minCardinality 10] [a owl:Restriction; owl:onProperty ex:isAuthorOf ; owl:someValuesFrom ex:Publication]) .`

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- (vii) **Left for exercises!**

OWL Example: A simple ontology about reviewers: What does it mean?

As we have seen, all these axioms boil down to first-order formulas, e.g.

- $ex:Publication \equiv ex:Paper \sqcap \exists ex:publishedIn. \top$ (iv)
- ```
ex:Publication owl:intersectionOf (ex:Paper [a owl:Restriction; owl:onProperty ex:publishedIn ;
owl:minCardinality 1]) .
```

Amounts to:

$$\forall x. Publication(x) \equiv Paper(x) \wedge \exists y. publishedIn(x, y)$$

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Amounts to:

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Recall, so far, when we wrote first-order, we always used a predicate *triple*:

$$\forall x. triples(x, a, ex:Publication) \equiv triples(x, a, ex:Publication) \wedge \exists y. triple(x, ex:publishedIn, y)$$

## Possible Problems (differences between OWL DL and OWL Full):

- classes/properties used as instances and vice versa (called “meta-modelling”)
- “faulty” OWL/RDF

⇒ Strictly speaking, if any of this occurs, I can't use my DL reasoner anymore, or at least it will be incomplete. :-)

# Possible Problems – faulty OWL/RDF

```
ex:Publication owl:intersectionOf (ex:Paper [a owl:Restriction; owl:onProperty ex:publishedIn ;
owl:minCardinality 1]) .
```

# Possible Problems – faulty OWL/RDF

```
ex:Publication owl:intersectionOf _:b1 . _:b1 rdfs:first _:b1 .
```

Has no meaning in the DL reading! One could just ignore such statements.



## Possible Problems – “meta-modelling”

```
ex:Publication owl:intersectionOf (ex:Paper [a owl:Restriction; owl:onProperty ex:publishedIn ;
owl:minCardinality 1]) .
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ex:Publication owl:intersectionOf (ex:Paper [a owl:Restriction; owl:onProperty ex:publishedIn ;
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```
ex:Paper owl:sameAs ex:publishedIn .
```

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```
ex:Paper owl:sameAs ex:publishedIn .
```

Unary/Binary first-order translation doesn't work anymore:

$$\forall x. \textit{Publication}(x) \equiv \textit{Paper}(x) \wedge \exists y \textit{publishedIn}(x, y)$$

$$\wedge \textit{Publication} = \textit{publishedIn}$$

Ouch! This is no longer first-order! And outside OWL DL...

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ex:Publication owl:intersectionOf (ex:Paper [a owl:Restriction; owl:onProperty ex:publishedIn ;
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Unary/Binary first-order translation doesn't work anymore:

$$\forall x. Publication(x) \equiv Paper(x) \wedge \exists y publishedIn(x, y)$$

$$\wedge Publication = publishedIn$$

Ouch! This is no longer first-order! And outside OWL DL...

*triple* encoding somewhat more stable here:

$$\begin{aligned} \forall x. triples(x, a, ex:Publication) \equiv \\ & triples(x, a, ex:Publication) \wedge \exists y triple(x, ex:publishedIn, y) \\ & \wedge triple(ex:Publication, owl:sameAs, ex:publishedIn) \end{aligned}$$

With the latter, I could still use a first-order reasoner for OWL Full.

Can I do OWL rules-based FWD-chaining inference as before?

Can I do OWL rules-based FWD-chaining inference as before?

- with some caveats, yes. . .
- inherently incomplete. . .
- . . . but some interesting subset of sound inferences might be covered

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Can I do OWL use other scalable techniques such as query rewriting?

- Yes! **OWL 2 QL**

→ More on that in Lecture 6 (OWL2 and OWL2 profiles)!

## Recommended Reading

- [ter Horst, 2005], RDF, RDFS and parts of OWL entailment in Rules.
- [Muñoz *et al.*, 2007], minimal sets of complete inference rules for parts of the RDFS vocabulary.

A bit more tough reading (specs), but also recommended:

- [Hayes, 2004, Sections 3 onwards], official RDF semantics specification.





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