Unit 3 – Higher Entailment regimes: Semantics of RDF(S), Datatypes, and OWL

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Unit Outline

1. RDF(S) Entailment: Giving semantics to the rdf: and rdfs: Vocabulary
2. D-Entailment: Giving Semantics to Datatypes
3. OWL Entailment: Giving Semantics to the owl: Vocabulary

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- ...That is: a semantics for the **rdf:**, **rdfs:** and **owl:** vocabularies

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- ...That is: a semantics for the **rdf:**, **rdfs:** and **owl:** vocabularies

Finally, how will we query inferred data?

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In the following, we use the following namespace prefixes:

```
@prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#> .
@prefix rdfs: <http://www.w3.org/2000/01/rdf-schema#> .
@prefix xsd: <http://www.w3.org/2001/XMLSchema#> .
@prefix owl: <http://www.w3.org/2002/07/owl#> .
```

The `rdf:` vocabulary

The RDF vocabulary, rdfV, is a set of URI references in the `rdf:` namespace:

- `rdf:type` ... expresses class membership
- `rdf:Property` ... the class of properties
- `rdf:XMLLiteral` ... the datatype of XML Literals.
- `rdf:first`, `rdf:rest`, `rdf:nil`, `rdf:List` ... vocabulary to write lists (called "collections") in RDF
- `rdf:Seq`, `rdf:Bag`, `rdf:Alt`, `rdf:_1`, `rdf:_2` ... vocabulary to describe (ordered, unordered, alternate) containers of values.
- `rdf:Statement`, `rdf:subject`, `rdf:predicate`, `rdf:object` ... vocabulary to make statements about statement (aka "reification")
- `rdf:value` ... to describe structured values

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- `rdf:value` ...to describe structured values

Note: The first three are the most important for the formal semntics. The semantics doesn't make any strong restrictions on the grayed out vocabulary.

Collection vocabulary – Example

Container vocab intended to be used to write lists, e.g.

```
ex:article ex:hasAuthors ( ex:johnny ex:jim ex:jack )
```

Collection vocabulary – Example

Container vocab intended to be used to write lists, e.g.

```
ex:article ex:hasAuthors _:b1.
_:b1 rdf:first ex:johnny ; rdf:rest _:b2.
_:b2 rdf:first ex:jim ; rdf:rest _:b3.
_:b3 rdf:first ex:jack ; rdf:rest rdf:nil.
```

Comes usually from RDF/XML's `parseType = "Collection"`.

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```

Comes usually from RDF/XML's `parseType = "Collection"`.

However, the semantics does not impose strong formal restrictions/conditions on the use of this vocabulary, e.g. a graph

```
ex:a rdf:first ex:a.
```

is fine (no restrictions on cyclic, incomplete, etc. lists.)

Container vocabulary – Example

Container vocab intended to be used to write containers of values, e.g.

```
ex:semWebCourse ex:hasParticipants [ a rdf:Bag;
                                     rdf:_1 ex:johny,
                                     rdf:_2 ex:jim;
                                     rdf:_3 ex:jack ] .
ex:semWebCourse ex:hasLecture [  a rdf:Alt;
                                 rdf:_1 rdf:axel
                                 rdf:_2 ex:thomas ] .
```

Comes usually from RDF/XML's `<li>...</li>`.

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Comes usually from RDF/XML's `<li>...</li>`.

However, the semantics does not impose strong formal restrictions/conditions on the use of this vocabulary, e.g. a graph.

```
ex:a rdf:_4 rdf:Seq;
     rdf:_3 rdf:_3 .
```

is fine (e.g. no restrictions on multiple occurrences of a numbered membership property, etc.).

Reification vocabulary & rdf:value – Example

Reification vocab intended to be used to write statements about statements, e.g.

```
ex:axel ex:thinks [ rdf:subject dbpedia:Web_Ontology_Language;
                    rdf:predicate owl:differentFrom
                    rdf:object dbpedia:Description_Logics] .
```

Again, no semantics restrictions on the use of this vocabulary.

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Again, no semantics restrictions on the use of this vocabulary.

`rdf:value` intended to be used to denote "structured values"

```
ex:telephone1 ex:hasPrice [ rdf:value 20; ex:currency ‘‘EUR’’]
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Again, no semantics restrictions on the use of this vocabulary.

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Purely speculative statement by the lecturer

Intuitively, all these parts of the vocabulary have been left without fixed semantics for the moment... may be changed by later standards? e.g. "Reification-Entailment", etc.

rdf-interpretation

An rdf-interpretation is a simple interpretation $I = \langle IR, IP, IEXT, IS, IL, LV \rangle$ which fulfills the following conditions and all the folllowing set of axiomatic triples A_{rdf}:

RDF semantic conditions.

x is in IP if and only if <x, I(`rdf:Property`)> is in IEXT(I(`rdf:type`))
If "xxx"`^^rdf:XMLLiteral` is in V and xxx is a well-typed XML literal string, then IL("xxx"`^^rdf:XMLLiteral`) is the XML value of xxx; IL("xxx"`^^rdf:XMLLiteral`) is in LV; IEXT(I(`rdf:type`)) contains <IL("xxx"`^^rdf:XMLLiteral`), I(`rdf:XMLLiteral`)>
If "xxx"`^^rdf:XMLLiteral` is in V and xxx is an ill-typed XML literal string, then IL("xxx"`^^rdf:XMLLiteral`) is not in LV; IEXT(I(`rdf:type`)) does not contain <IL("xxx"`^^rdf:XMLLiteral`), I(`rdf:XMLLiteral`)>

RDF axiomatic triples.

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rdf:type rdf:type rdf:Property .
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rdf:first rdf:type rdf:Property .
rdf:rest rdf:type rdf:Property .
rdf:value rdf:type rdf:Property .
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...
rdf:nil rdf:type rdf:List .
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```

rdf-satisaction, entailment

written $\models_{rdf}$ defined analogously to $\models$, but with respect to rdf-interpretations.

rdf-interpretation

Some consequences:

Proposition 1

If `s p o.` $\in G$ then $G \models_{rdf}$ {`p a rdf:Property`}.

Particularly, note that the first axiomatic triple is redundant!

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Let L_{XML} denote the set of well-formed XML literals.

Proposition 2

If `s p "`xyz`"^^rdf:XMLLiteral.` $\in G$, such that $xyz \in L_{XML}$ then $G \models_{rdf}$ {`[] a rdf:XMLLiteral`}.

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Remark: ill-formed XML literals are hardly (impossibly?) writeable in RDF/XML, but in other syntaxes.

Computing entailment for $\models_{rdf}$

Recall main result for simple entailment ($\models$):

Interpolation Lemma

$S \models E$ if and only if a subgraph of S is an instance of E.

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Does no longer hold for $\models_{rdf}$:-(

BUT! Can be enabled by rules!

rdf-closure ($Cl_{rdf}(G)$)

The rdf-closure $Cl_{rdf}(G)$ of graph G is defined by $G \uplus A_{rdf}$ plus exhaustive application of the following rules to G:

- If `s p o.` $\in Cl_{rdf}(G)$ then add `p a rdf:Property.`
- If `s p "`xyz`"^^rdf:XMLLiteral.` $\in Cl_{rdf}(G)$ and $xyz \in L_{XML}$ then add `_:xyz a rdf:XMLLiteral.` where _:xyz is a distinct bnode for that literal.

Slight abstraction of derivation rules $rdf1$ and $rdf2$ from the spec.

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- p `a rdf:Property .` $\Leftarrow s\ p\ o$.
- `_:xyz a rdf:XMLLiteral .` $\Leftarrow s\ p$ "xyz"^^`rdf:XMLLiteral.`

Let's write them in more "rule" syntax.

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- $\forall s, p, o(triple(p, \texttt{rdf:type}, \texttt{rdf:Property}) \leftarrow triple(s, p, o))$
- $\forall s, p \exists b(triple(b, \texttt{rdf:type}, \texttt{rdf:XMLLiteral}) \leftarrow triple(s, p, \texttt{"}xyz\texttt{"}\verb|^^|\texttt{rdf:XMLLiteral}))$

Or as first-order sentences.

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- `triple(P,rdf:type,rdf:Property) :- triple(S,P,O).`
- `triple(`$\mathtt{b}_{xyz}$`,rdf:type,rdf:XMLLiteral) :- triple(S,P,"`xyz`"^^rdf:XMLLiteral)`

Or as Datalog rules.

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RDF entailment lemma (slightly adapted from the spec)

$S \models_{rdf} E$ if and only if $Cl_{rdf}(S) \models E$

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But: A finite subset of $Cl_{rdf}(G)$ is sufficient! Let reduced closure of G_1 w.r.t. G_2, written $Cl_{rdf}(G_1, G_2)$, be defined as $Cl_{rdf}(G_1)$ with only those axiomatic triples `rdf:_`n `rdf:type rdf:Property .` where `rdf:_`$n \in V(G_2)$

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$\Rightarrow$ For a finite graph G_2, $Cl_{rdf}(G_1, G_2)$ is always finite.

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Means that we can compute RDF entailment finitely! :-)

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...Didn't buy us too much, except that:

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- we have a way to express that a predicate is an `rdf:Propery`
- we have a way to express that XML Literals are `rdf:XMLLiterals`

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- we have a way to express that a predicate is an `rdf:Propery`
- we have a way to express that XML Literals are `rdf:XMLLiterals`

Still, we cannot **describe other vocabularies** $\Rightarrow$ rdfs:

The rdf: vocabulary

The RDFS vocabulary, rdfsV, extends rdfV:

- `rdfs:domain`, `rdfs:range`
- `rdfs:subClassOf`, `rdfs:subPropertyOf`
- `rdfs:Resource` . . . all resources.
- `rdfs:Class` . . . all resources which denote classes.
- `rdfs:Literal` . . . the “class of literal values”.
- `rdfs:Datatype` . . . the “class of datatypes”.
- `rdfs:Container` . . . a joint superclass of `rdf:Alt`, `rdf:Seq`, `rdf:Bag`.
- `rdfs:ContainerMembershipProperty` . . . the “class of conainer membership predicates” (`rdf:_1`, `rdf:_2`,. . .).
- `rdfs:member` a joint superproperty of `rdf:_1`, `rdf:_2`,. . . .
- `rdfs:comment`, `rdfs:seeAlso`, `rdfs:isDefinedBy`, `rdfs:label` some more conventional vocabulary without strong “formal” semantics[1]

[1] for illustrative examples, let’s check the FOAF ontology
RDF/XML version: `http://xmlns.com/foaf/spec/20071002.rdf`
Extracted Turtle version: `http://www.polleres.net/teaching/SemWebTech_2009/testdata/foafOntology.ttl`

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Additionally to IR and IP, the semantic conditions for RDF also talk about the set of classes, denoted by $IC \subseteq IR$:

IC

IC is defined as the set of resources which appear in the range of the extension of `rdf:type`, i.e. all the y such that $\langle x, y\rangle \in IEXT(I(\texttt{rdf:type}))$.

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For brevity, we also define:

$ICEXT$

$ICEXT : IC \rightarrow 2^{IR}$ defined as $x \in ICEXT(y) \Leftrightarrow \langle x, y\rangle \in IEXT(I(\texttt{rdf:type}))$

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Let's look into the semantic conditions and axiomatic triples for rdfs-interpretations now...

rdfs-interpretations: Semantic conditions

x is in ICEXT(y) if and only if <x,y> is in IEXT(I(`rdf:type`))

IC = ICEXT(I(`rdfs:Class`))

IR = ICEXT(I(`rdfs:Resource`))

LV = ICEXT(I(`rdfs:Literal`))

If <x,y> is in IEXT(I(`rdfs:domain`)) and <u,v> is in IEXT(x) then u is in ICEXT(y)

If <x,y> is in IEXT(I(`rdfs:range`)) and <u,v> is in IEXT(x) then v is in ICEXT(y)

IEXT(I(`rdfs:subPropertyOf`)) is transitive and reflexive on IP

If <x,y> is in IEXT(I(`rdfs:subPropertyOf`)) then x and y are in IP and IEXT(x) is a subset of IEXT(y)

If x is in IC then <x, I(`rdfs:Resource`)> is in IEXT(I(`rdfs:subClassOf`))

If <x,y> is in IEXT(I(`rdfs:subClassOf`)) then x and y are in IC and ICEXT(x) is a subset of ICEXT(y)

IEXT(I(`rdfs:subClassOf`)) is transitive and reflexive on IC

If x is in ICEXT(I(`rdfs:ContainerMembershipProperty`)) then:
< x, I(`rdfs:member`)> is in IEXT(I(`rdfs:subPropertyOf`))

If x is in ICEXT(I(`rdfs:Datatype`)) then <x, I(`rdfs:Literal`)> is in IEXT(I(`rdfs:subClassOf`))

rdfs-interpretations: Semantic conditions

x is in ICEXT(y) if and only if <x,y> is in IEXT(I(`rdf:type`))

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o `a rdf:Class .` $\Leftarrow$ s `a` o `.`
`[] a` s `.` $\Leftarrow$ s `a rdf:Class .`
s `a rdf:Resource .` $\Leftarrow$ $s\ p\ o$ `.`
p `a rdf:Resource .` $\Leftarrow$ $s\ p\ o$ `.`
o `a rdf:Resource .` $\Leftarrow$ $s\ p\ o$ `.` $o \notin L$
`_:l a rdf:Literal .` $\Leftarrow$ $s\ p\ l$. $l \in L$
$s\ p$ `_:l .` $\Leftarrow$ $s\ p\ l$ `.` $l \in L$

If <x,y> is in IEXT(I(`rdfs:domain`)) and <u,v> is in IEXT(x) then u is in ICEXT(y)

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IEXT(I(`rdfs:subClassOf`)) is transitive and reflexive on IC

If x is in ICEXT(I(`rdfs:ContainerMembershipProperty`)) then:
< x, I(`rdfs:member`)> is in IEXT(I(`rdfs:subPropertyOf`))

If x is in ICEXT(I(`rdfs:Datatype`)) then <x, I(`rdfs:Literal`)> is in IEXT(I(`rdfs:subClassOf`))

rdfs-interpretations: Semantic conditions

x is in ICEXT(y) if and only if <x,y> is in IEXT(I(`rdf:type`))

IC = ICEXT(I(`rdfs:Class`))

IR = ICEXT(I(`rdfs:Resource`))

LV = ICEXT(I(`rdfs:Literal`))

o a rdf:Class . $\Leftarrow$ s a o .
[] a s . $\Leftarrow$ s a rdf:Class .
s a rdf:Resource . $\Leftarrow$ s p o .
p a rdf:Resource . $\Leftarrow$ s p o .
o a rdf:Resource . $\Leftarrow$ s p o . $o \notin L$
_:l a rdf:Literal . $\Leftarrow$ s p l. $l \in L$
s p _:l . $\Leftarrow$ s p l . $l \in L$

If <x,y> is in IEXT(I(`rdfs:domain`)) and <u,v> is in IEXT(x) then u is in ICEXT(y)

s a c . $\Leftarrow$ p domain c . s p o .

If <x,y> is in IEXT(I(`rdfs:range`)) and <u,v> is in IEXT(x) then v is in ICEXT(y)

IEXT(I(`rdfs:subPropertyOf`)) is transitive and reflexive on IP

If <x,y> is in IEXT(I(`rdfs:subPropertyOf`)) then x and y are in IP and IEXT(x) is a subset of IEXT(y)

If x is in IC then <x, I(`rdfs:Resource`)> is in IEXT(I(`rdfs:subClassOf`))

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o `a rdf:Class .` $\Leftarrow$ s `a` o `.`
`[] a` s `.` $\Leftarrow$ s `a rdf:Class .`
s `a rdf:Resource .` $\Leftarrow$ $s\ p\ o$ `.`
p `a rdf:Resource .` $\Leftarrow$ $s\ p\ o$ `.`
o `a rdf:Resource .` $\Leftarrow$ $s\ p\ o$ `.` $o \notin L$
`_:l a rdf:Literal .` $\Leftarrow$ $s\ p\ l$. $l \in L$
$s\ p$ `_:l .` $\Leftarrow$ $s\ p\ l$ `.` $l \in L$

If <x,y> is in IEXT(I(`rdfs:domain`)) and <u,v> is in IEXT(x) then u is in ICEXT(y)

s `a` c `.` $\Leftarrow$ p `domain` c `.` $s\ p\ o$ `.`

If <x,y> is in IEXT(I(`rdfs:range`)) and <u,v> is in IEXT(x) then v is in ICEXT(y)

o `a` c `.` $\Leftarrow$ p `range` c `.` $s\ p\ o$ `.` $o \notin L$

IEXT(I(`rdfs:subPropertyOf`)) is transitive and reflexive on IP

If <x,y> is in IEXT(I(`rdfs:subPropertyOf`)) then x and y are in IP and IEXT(x) is a subset of IEXT(y)

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LV = ICEXT(I(`rdfs:Literal`))

- o `a rdf:Class .` $\Leftarrow s$ `a` o `.`
- `[] a` s `.` $\Leftarrow s$ `a rdf:Class .`
- s `a rdf:Resource .` $\Leftarrow s\ p\ o$ `.`
- p `a rdf:Resource .` $\Leftarrow s\ p\ o$ `.`
- o `a rdf:Resource .` $\Leftarrow s\ p\ o$ `.` $\quad o \notin L$
- `_:l a rdf:Literal .` $\Leftarrow s\ p\ l$. $\quad l \in L$
- $s\ p$ `_:l .` $\Leftarrow s\ p\ l$ `.` $\quad l \in L$

If <x,y> is in IEXT(I(`rdfs:domain`)) and <u,v> is in IEXT(x) then u is in ICEXT(y)

- s `a` c `.` $\Leftarrow p$ `domain` c `.` $s\ p\ o$ `.`

If <x,y> is in IEXT(I(`rdfs:range`)) and <u,v> is in IEXT(x) then v is in ICEXT(y)

- o `a` c `.` $\Leftarrow p$ `range` c `.` $s\ p\ o$ `.` $\quad o \notin L$

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- s `subPropertyOf` r `.` $\Leftarrow s$ `subPropertyOf` o `.` o `subPropertyOf` r `.`
- s `subPropertyOf` s `.` $\Leftarrow s$ `a Property .`
- s `a Property .` $\Leftarrow s$ `subPropertyOf` o `.`
- o `a Property .` $\Leftarrow s$ `subPropertyOf` o `.`
- $s\ q\ o \Leftarrow p$ `subPropertyOf` q. $s\ p\ o$. $\quad q \notin BL$
- s `subClassOf Resource .` $\Leftarrow s$ `a Class .`
- s `a Class .` $\Leftarrow s$ `subClassOf` o `.`
- o `a Class .` $\Leftarrow s$ `subClassOf` o `.` $\quad o \notin L$
- s `a` c `.` $\Leftarrow o$ `suClassOf` c `.` s `a` o `.`
- s `subClassOf` r `.` $\Leftarrow s$ `subClassOf` o `.` o `subClassOf` r `.`
- s `subClassOf` s `.` $\Leftarrow s$ `a Property .`
- s `subPropertyOf member .`$\Leftarrow$ s `a ContainerMembershipProperty.`
- s `subClassOf Literal .` $\Leftarrow s$ `a Datatype.`

rdfs-interpretations: Additional Axiomatic Triples

A_{rdfs}

```
rdf:type            rdfs:domain rdfs:Resource;
                    rdfs:range rdfs:Class .
rdfs:domain         rdfs:domain rdf:Property;
                    rdfs:range rdfs:Class .
rdfs:range          rdfs:domain rdf:Property;
                    rdfs:range rdfs:Class .
rdfs:subPropertyOf  rdfs:domain rdf:Property;
                    rdfs:range  rdf:Property .
rdfs:subClassOf     rdfs:domain rdfs:Class;
                    rdfs:range  rdfs:Class .

rdfs:member       rdfs:domain rdfs:Resource;
                  rdfs:range rdfs:Resource .
rdfs:seeAlso      rdfs:domain rdfs:Resource;
                  rdfs:range rdfs:Resource .
rdfs:isDefinedBy  rdfs:domain rdfs:Resource;
                  rdfs:range rdfs:Resource .
rdf:value         rdfs:domain rdfs:Resource;
                  rdfs:range rdfs:Resource .

rdfs:comment      rdfs:domain rdfs:Resource;
                  rdfs:range rdfs:Literal .
rdfs:label        rdfs:domain rdfs:Resource;
                  rdfs:range rdfs:Literal .

rdf:first         rdfs:domain rdf:List;
                  rdfs:range rdfs:Resource .
rdf:rest          rdfs:domain rdf:List .
                  rdfs:range rdf:List .
```

```
rdf:subject      rdfs:domain rdf:Statement;
                 rdfs:range rdfs:Resource .
rdf:predicate    rdfs:domain rdf:Statement;
                 rdfs:range rdfs:Resource .
rdf:object       rdfs:domain rdf:Statement;
                 rdfs:range rdfs:Resource .

rdf:Alt rdfs:subClassOf rdfs:Container .
rdf:Bag rdfs:subClassOf rdfs:Container .
rdf:Seq rdfs:subClassOf rdfs:Container .
rdfs:ContainerMembershipProperty
        rdfs:subClassOf rdf:Property .

rdfs:isDefinedBy rdfs:subPropertyOf rdfs:seeAlso .

rdf:XMLLiteral rdf:type rdfs:Datatype .
rdf:XMLLiteral rdfs:subClassOf rdfs:Literal .
rdfs:Datatype rdfs:subClassOf rdfs:Class .

rdf:_1 rdf:type rdfs:ContainerMembershipProperty .
rdf:_1 rdfs:domain rdfs:Resource .
rdf:_1 rdfs:range rdfs:Resource .
rdf:_2 rdf:type rdfs:ContainerMembershipProperty .
rdf:_2 rdfs:domain rdfs:Resource .
rdf:_2 rdfs:range rdfs:Resource .
...
```

rdfs-interpretations: Axiomatic Triples

A_{rdfs} – "somewhat redundant triples" (let's call that A^{-}_{rdfs})

```
rdf:type            rdfs:range rdfs:Class .
rdfs:domain         rdfs:range rdfs:Class .
rdfs:range          rdfs:range rdfs:Class .

rdfs:subClassOf     rdfs:domain rdfs:Class;
                    rdfs:range  rdfs:Class .

rdfs:comment     rdfs:range rdfs:Literal .
rdfs:label       rdfs:range rdfs:Literal .

rdf:first        rdfs:domain rdf:List .
rdf:rest         rdfs:domain rdf:List .
                 rdfs:range rdf:List .
```

```
rdf:subject      rdfs:domain rdf:Statement;
rdf:predicate    rdfs:domain rdf:Statement;
rdf:object       rdfs:domain rdf:Statement;

rdf:Alt rdfs:subClassOf rdfs:Container .
rdf:Bag rdfs:subClassOf rdfs:Container .
rdf:Seq rdfs:subClassOf rdfs:Container .
rdfs:ContainerMembershipProperty
        rdfs:subClassOf rdf:Property .

rdfs:isDefinedBy rdfs:subPropertyOf rdfs:seeAlso .

rdf:XMLLiteral rdf:type rdfs:Datatype .
rdf:XMLLiteral rdfs:subClassOf rdfs:Literal .
rdfs:Datatype rdfs:subClassOf rdfs:Class .

rdf:_1 rdf:type rdfs:ContainerMembershipProperty .
rdf:_2 rdf:type rdfs:ContainerMembershipProperty .
...
```

Actually, one could think: A^{-}_{rdfs} is enough:

- Axiomatic triples s `domain/range rdfs:Resource .`
- Axiomatic triples p `domain/range rdf:Property .`

rdfs-interpretations: Axiomatic Triples

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```
rdf:type          rdfs:range rdfs:Class .
rdfs:domain       rdfs:range rdfs:Class .
rdfs:range        rdfs:range rdfs:Class .

rdfs:subClassOf   rdfs:domain rdfs:Class;
                  rdfs:range  rdfs:Class .

rdfs:comment    rdfs:range rdfs:Literal .
rdfs:label      rdfs:range rdfs:Literal .

rdf:first       rdfs:domain rdf:List .
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rdf:subject      rdfs:domain rdf:Statement;
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rdf:Seq rdfs:subClassOf rdfs:Container .
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rdfs:isDefinedBy rdfs:subPropertyOf rdfs:seeAlso .

rdf:XMLLiteral rdf:type rdfs:Datatype .
rdf:XMLLiteral rdfs:subClassOf rdfs:Literal .
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...
```

Actually, one could think: A^{-}_{rdfs} is enough:

- Axiomatic triples s `domain/range rdfs:Resource .`
- Axiomatic triples p `domain/range rdf:Property .`

More on that later. . . ... bottomline: if we aren't interested in inferring triples

x `rdfs:domain rdfs:Resource.` , we can probably do without these.

rdfs-entailment: Properties

Recall:

Modified RDF entailment lemma (slightly adapted from the spec)

$S \models_{rdf} E$ if and only if $Cl_{rdf}(S, E) \models E$

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. . . but, means that we can compute RDFS entailment finitely! :-)

Spec entailment rules are – incomplete!

Some of the rules we had before had restrictions in the antecedent, e.g.

s **a** c . $\Leftarrow p$ `domain` c . $s\ p\ o$.
$s\ q\ o \Leftarrow p$ `subPropertyOf` q. $s\ p\ o$. $q \notin BL$

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But what about that:

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p rdfs:subPropertyOf _:p
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s p o .
```

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Problem: If we execute the rules with the restrictions in place, we won't get:

```
s rdf:type c.
```

The invalid "intermediate" triple

```
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```

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p rdfs:subPropertyOf _:p
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s p o .
```

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```
s rdf:type c.
```

The invalid "intermediate" triple

```
s _:p c.
```

cannot be inferred.

Solution: easy, (i) allow "generalized" RDF ($UBL \times UBL \times UBL$) for intermediate computation of closure, or (ii) use modified "implicit typing rule", cf. [Muñoz *et al.*, 2007]:

s **a** $c \Leftarrow p$ `subPropertyOf` q. q `rdfs:domain` c. $s\ p\ o$.

Inconsistency

Proposition 3

ANY Graph

- containing a triple s p `"`xyz`"^^rdf:XMLLiteral.` such that xyz is NOT a well-formed XML literal and
- allowing to infer the triple p `rdf:range rdf:XMLLiteral .`

is false in any rdfs-interpretation.

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Why?

- by semantic conditions of rdf- and rdfs-inerpretations!

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- allowing to infer the triple p `rdf:range rdf:XMLLiteral .`

is false in any rdfs-interpretation.

Why?

- by semantic conditions of rdf- and rdfs-inerpretations!

Implication:

- There can be rdfs-inconsistent RDF graphs, e.g.

```
ex:a ex:p "<notLegalXML"^^rdf:XMLLiteral .
ex:p rdfs:range rdf:XMLLiteral .
```

rdfs-entailment: Example 1

$\{\} \models_{rdfs} G_5$

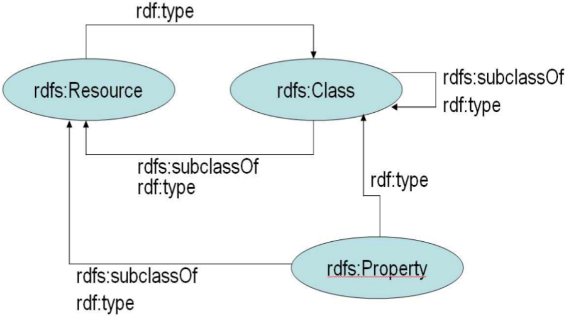
G_5 :

```
rdfs:Resource rdf:type rdfs:Class.
rdfs:Property rdf:type rdfs:Resource.
rdfs:Property rdf:subclassOf rdfs:Resource.
rdfs:Property rdf:type rdfs:Class.
rdfs:Class rdf:type rdfs:Resource.
rdfs:Class rdf:type rdfs:Class.
rdfs:Class rdf:subclassOf rdfs:Resource.
rdfs:Class rdf:subclassOf rdfs:Class.
```

rdfs-entailment: Example 1

$\{\} \models_{rdfs} G_5$

G_5 :



rdfs-entailment: Example 2

$G_1 \models_{rdfs} G_2$

G_1 :

```
ex:alice foaf:knows ex:bob.
ex:alice foaf:name "Alice".
foaf:knows rdfs:domain foaf:Person.
```

G_2 :

```
ex:alice rdf:type foaf:Person.
```

rdfs-entailment: Example 3

$\{\} \models_{rdfs}$ { xxx `rdf:type rdfs:Resource.` }

for all xxx in the Vocabulary of any interpretation I.

Implication: Since all rdfs-Interpretations are over an infinite vocabulary (`rdf:_`n), even without the axiomatic triples that would already entail infinite triples.

Unit Outline

1. RDF(S) Entailment: Giving semantics to the rdf: and rdfs: Vocabulary
2. D-Entailment: Giving Semantics to Datatypes
3. OWL Entailment: Giving Semantics to the owl: Vocabulary

Datatype

A datatype d is an entity characterized by a set of character strings called lexical forms (or *lexical space*) and a mapping from that set to a set of values (called *value space*). Exactly how these sets and mappings (called lexical-to-value mapping $L2V(d)$) are defined is a matter external to RDF, e.g. XML Schema [XML Schema Datatypes, 2001].

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A *datatype map* is a set of datatypes.

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A *datatype map* is a set of datatypes.

Besides `rdf:XMLLiteral`, the RDF spec suggests to support the following XML Schema datatypes:

XSD datatypes
xsd:string, xsd:boolean, xsd:decimal, xsd:float, xsd:double, xsd:dateTime, xsd:time, xsd:date, xsd:gYearMonth, xsd:gYear, xsd:gMonthDay, xsd:gDay, xsd:gMonth, xsd:hexBinary, xsd:base64Binary, xsd:anyURI, xsd:normalizedString, xsd:token, xsd:language, xsd:NMTOKEN, xsd:Name, xsd:NCName, xsd:integer, xsd:nonPositiveInteger, xsd:negativeInteger, xsd:long, xsd:int, xsd:short, xsd:byte, xsd:nonNegativeInteger, xsd:unsignedLong, xsd:unsignedInt, xsd:unsignedShort, xsd:unsignedByte, xsd:positiveInteger

D-interpretations

D-interpretation

A D-interpretation is an rdfs-interpretation I defined by the following additional semantic conditions:

General semantic conditions for datatypes.

if <aaa,x> is in D then I(aaa) = x
if <aaa,x> is in D then ICEXT(x) is the value space of x and is a subset of LV
if <aaa,x> is in D then for any typed literal "sss"^^ddd in V with I(ddd) = x , if sss is in the lexical space of x then IL("sss"^^ddd) = L2V(x)(sss), otherwise IL("sss"^^ddd) is not in LV
if <aaa,x> is in D then I(aaa) is in ICEXT(I(`rdfs:Datatype`))

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if <aaa,x> is in D then I(aaa) is in ICEXT(I(`rdfs:Datatype`))

- In principle, just generalization of the Semantic conditions for `rdf:XMLLiteral` to other datatypes.
- Inuitive Reading: fix the interpretation of typed literals according to $L2V(d)$

That's it!

D-entailment ($\models_D$) – Examples

{ s p `"2.0"^^xsd:decimal.` } $\models_D$ { s p `"2"^^xsd:integer.` }

{ s p `"2"^^xsd:integer.` } $\models_D$ { s p `"2.0"^^xsd:decimal.` }

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{ s p `"2"^^xsd:integer.` } $\models_D$ { s p `"2.0"^^xsd:decimal.` }

D-inconsistent graphs:

`{ ex:a ex:b "25"^^xsd:decimal . ex:b rdfs:range xsd:string .}`
(because the value spaces of `sd:string` and `xsd:decimal` are disjoint)

`ex:a ex:p "2.5"^^xsd:decimal . ex:p rdfs:range xsd:integer .}`
(overlapping value spaces, but 2.5 outside `xsds:integer`'s value space)

`{ ex:a ex:b "haha"^^xsd:decimal. }`

(outside lexical space, i.e. "ill-formed" for `xsd:decimal`)

Rule-based computation of D-entailment?

- Basically, needs an oracle to evaluate L2V...

e.g. could be done by a built-in that maps all typed literals in a graph to compute $L2V$ of their value:

$s \ \ p \ \ L2V(l) \ \ . \Leftarrow s \ \ p \ \ l\text{^^}d \ \ .$ for l in the lex. space of d.

Rule-based computation of D-entailment?

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Again needs "generalized intermediate triples" $(UBL \cup Vs \times UBL \cup Vs \times UBL \cup Vs)$ for intermediate computation of closure, where Vs is the union of the value spaces of all supported datatypes.

Unit Outline

1. RDF(S) Entailment: Giving semantics to the rdf: and rdfs: Vocabulary
2. D-Entailment: Giving Semantics to Datatypes
3. OWL Entailment: Giving Semantics to the owl: Vocabulary

Semantics of the owl:Vocabulary

Not part of the RDF Spec, separate document [Patel-Schneider *et al.*, 2004].

- OWL adds the possiblity to add some DL axioms (TBox, ABox) to RDF ($\mathcal{SHOIN}(\mathcal{D})$).
- OWL defines a rewriting how to write such DL axioms in RDF.
- OWL adds the possiblity to add some DL statements to an ontology.
- Defines two semantics:
    - DL-style model-theoretic semantics, for the syntactic subset of OWL which corresponds to DL
    - RDF compatible semantics

OWL DL in two slides: 1/2

Expressing property characteristics:

OWL property axioms as RDF triples	DL syntax	FOL short representation
P `rdfs:domain` C .	$\top \sqsubseteq \forall P^{-}.C$	$\forall x, y.P(x, y) \supset C(x)$
P `rdfs:range` C .	$\top \sqsubseteq \forall P.C$	$\forall x, y.P(x, y) \supset C(y)$
P `owl:inverseOf` P_0 .	$P \equiv P_0^{-}$	$\forall x, y.P(x, y) \equiv P_0(y, x)$
P `rdf:type owl:SymmetricProperty`.	$P \equiv P^{-}$	$\forall x, y.P(x, y) \equiv P(y, x)$
P `rdf:type owl:FunctionalProperty`.	$\top \sqsubseteq \leqslant 1P$	$\forall x, y, z.P(x, y) \wedge P(x, z) \supset y = z$
P `rdf:type owl:InverseFunctionalProperty`.	$\top \sqsubseteq \leqslant 1P^{-}$	$\forall x, y, z.P(x, y) \wedge P(z, y) \supset x = z$
P `rdf:type owl:TransitiveProperty`.	$P^{+} \sqsubseteq P$	$\forall x, y, z.P(x, y) \wedge P(y, z) \supset P(x, z)$

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Expressing complex class descriptions:

OWL complex class descriptions*	DL syntax	FOL short representation
`owl:Thing`	$\top$	$x = x$
`owl:Nothing`	$\bot$	$\neg x = x$
`owl:intersectionOf` $(C_1 \ldots C_n)$	$C_1 \sqcap \cdots \sqcap C_n$	$C_1(x) \wedge \cdots \wedge C_n(x)$
`owl:unionOf` $(C_1 \ldots C_n)$	$C_1 \sqcup \cdots \sqcup C_n$	$C_1(x) \vee \cdots \vee C_n(x)$
`owl:complementOf` (C)	$\neg C$	$\neg C(x)$
`owl:oneOf` $(o_1 \ldots o_n)$	$\{o_1, \ldots, o_n\}$	$x = o_1 \vee \cdots \vee x = o_n$
`owl:restriction` $(P$ `owl:someValuesFrom` $(C))$	$\exists P.C$	$\exists y.P(x, y) \wedge C(y)$
`owl:restriction` $(P$ `owl:allValuesFrom` $(C))$	$\forall P.C$	$\forall y.P(x, y) \supset C(y)$
`owl:restriction` $(P$ `owl:value` $(o))$	$\exists P.\{o\}$	$P(x, o)$
`owl:restriction` $(P$ `owl:minCardinality` $(n))$	$\geqslant nP$	$\exists y_1 \ldots y_n. \bigwedge_{k=1}^{n} P(x, y_k) \wedge \bigwedge_{i<j} y_j \neq y_j$
`owl:restriction` $(P$ `owl:maxCardinality` $(n))$	$\leqslant nP$	$\forall y_1 \ldots y_{n+1}. \bigwedge_{k=1}^{n+1} P(x, y_k) \supset \bigvee_{i<j} y_i = y_j$

*For reasons of legibility, we use a variant of the OWL abstract syntax [Patel-Schneider *et al.*, 2004] in this table.

OWL DL in two slides: 2/2

Relating Class descriptions:

C_1 `rdfs:subClassOf` C_1	$C_1 \sqsubseteq C_2$
C_1 `owl:equivalentClass` C_2	$C_1 \equiv C_2$
C_1 `owl:disjointWith` C_2	$C_1 \sqcap C_2 \sqsubseteq \bot$

Relating individuals:

o_1 `owl:sameAs` o_1	$o_1 = o_2$
o_1 `owl:differentFrom` o_2	$o_1 \neq o_2$

OWL DL in two slides: 2/2

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Let's look into some examples of

- How to write OWL in RDF.
- what it means
- possible problems

How to write OWL in RDF – A simple ontology about reviewers:

- **Properties:** `title`, `isAuthorOf`, `publishedIn`, etc.
- **Classes:** `Senior`, `Paper`, `Publication`, etc.
- **Relations:**
  - *A `Publication` is a `Paper` which has been `published`* (subclass + existential condition on property)
  - *`isAuthorOf` is the opposite of Dublin Core's `dc:creator` Property*[2]
  - *A `Senior` researcher is a `foaf:Person` who `isAuthorOf` 10+ `Publications`* (subclass + condition on cardinality)
  - *Each item can be `publishedIn` at most one venue* (functional property)

$\vdots$

[2] reuse of external ontologies!

OWL Example: A simple ontology about reviewers: in DL

$$\exists ex{:}title.\top \sqsubseteq ex{:}Paper \tag{i}$$

$$\exists ex{:}title^{-}.\top \sqsubseteq xsd{:}string \tag{ii}$$

$$ex{:}isAuthorOf^{-} \equiv dc{:}creator \tag{iii}$$

$$ex{:}Publication \equiv ex{:}Paper \sqcap \exists ex{:}publishedIn.\top \tag{iv}$$

$$\top \sqsubseteq \leqslant 1 \; ex{:}publishedIn^{-} \tag{v}$$

$$\begin{aligned} ex{:}Senior \equiv foaf{:}Person \sqcap \geqslant 10 \; ex{:}isAuthorOf \sqcap \\ \exists ex{:}isAuthorOf.ex{:}Publication \end{aligned} \tag{vi}$$

$$ex{:}Club100 \equiv foaf{:}Person \sqcap \geqslant 100 \; ex{:}isAuthorOf \tag{vii}$$

OWL Example: A simple ontology about reviewers: in OWL/RDF

- $\exists ex{:}title.\top \sqsubseteq ex{:}Paper$ (i)

OWL Example: A simple ontology about reviewers: in OWL/RDF

- $\exists ex{:}title.\top \sqsubseteq ex{:}Paper$ (i)
  ```
  [a owl:Restriction; owl:onProperty ex:title; owl:someValuesFrom owl:Thing]
  rdfs:subclassOf ex:Paper .
  ```

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  `owl:Thing rdfs:subclassOf [ a owl:restriction; on Property ex:publishedIn; owl:maxCardinality 1 ].`

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- *A `Senior` researcher is a `foaf:Person` who `isAuthorOf` 10+ `Publications`* (vi)

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  `ex:publishedIn a owl:FunctionalProperty .`
- *A* `Senior` *researcher is a* `foaf:Person` *who* `isAuthorOf` *10+* `Publications` (vi)
  `ex:Senior owl:intersectionOf ( foaf:Person [ a owl:Restriction; owl:onProperty ex:isAuthorOf ; owl:minCardinality 10 ] [ a owl:Restriction; owl:onProperty ex:isAuthorOf ; owl:someValuesFrom ex:Publication ] ) .`

OWL Example: A simple ontology about reviewers: in OWL/RDF

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- (vii) **Left for exercises!**

OWL Example: A simple ontology about reviewers: What does it mean?

As we have seen, all these axioms boil down to first-order formulas, e.g.

- $ex{:}Publication \equiv ex{:}Paper \sqcap \exists ex{:}publishedIn.\top$ (iv)
  `ex:Publication owl:intersectionOf ( ex:Paper [ a owl:Restriction; owl:onProperty ex:publishedIn ; owl:minCardinality 1 ] ) .`

Amounts to:

$$\forall x.Publication(x) \equiv Paper(x) \land \exists y publishedIn(x, y)$$

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Amounts to:

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Recall, so far, when we wrote first-order, we always used a predicate $triple$:

$$\forall x.triples(x, \mathtt{a}, \mathtt{ex:Publication}) \equiv triples(x, \mathtt{a}, \mathtt{ex:Publication}) \wedge \exists y triple(x, \mathtt{ex:publishedIn}, y)$$

Possible Problems (differences between OWL DL and OWL Full):

- classes/properties used as instances and vice versa (called "meta-modelling")
- "faulty" OWL/RDF

$\Rightarrow$ Strictly speaking, if any of this occurs, I can't use my DL reasoner anymore, or at least it will be incomplete. :-(

Possible Problems – faulty OWL/RDF

```
ex:Publication owl:intersectionOf ( ex:Paper [ a owl:Restriction; owl:onProperty ex:publishedIn ;
owl:minCardinality 1 ] ) .
```

Possible Problems – faulty OWL/RDF

```
ex:Publication owl:intersectionOf _:b1 .  _:b1 rdfs:first _:b1 .
```

Has no meaning in the DL reading! One could just ignore such statements.

Possible Problems – “meta-modelling”

```
ex:Publication owl:intersectionOf ( ex:Paper [ a owl:Restriction; owl:onProperty ex:publishedIn ;
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Possible Problems – "meta-modelling"

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ex:Publication owl:intersectionOf ( ex:Paper [ a owl:Restriction; owl:onProperty ex:publishedIn ;
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exPaper owl:sameAs ex:publishedIn .

Possible Problems – "meta-modelling"

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exPaper owl:sameAs ex:publishedIn .

Unary/Binary first-order translation doesn't work anymore:

$$\forall x.Publication(x) \equiv Paper(x) \land \exists y publishedIn(x, y)$$

$$\land Publication = publishedIn$$

Ouch! This is no longer first-order! And outside OWL DL...

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$$\wedge Publication = publishedIn$$

Ouch! This is no longer first-order! And outside OWL DL...
triple encoding somewhat more stable here:

$$\begin{aligned}&\forall x.triples(x, \mathtt{a}, \mathtt{ex:Publication}) \equiv \\ &\quad triples(x, \mathtt{a}, \mathtt{ex:Publication}) \wedge \exists y triple(x, \mathtt{ex:publishedIn}, y) \\ &\quad \wedge triple(\mathtt{ex:Publication}, \mathtt{owl:sameAs}, \mathtt{ex:publishedIn}\end{aligned}$$

With the latter, I could still use a first-order reasoner for OWL Full.

Can I do OWL rules-based FWD-chaining inference as before?

Can I do OWL rules-based FWD-chaining inference as before?

- with some caveats, yes...
- inherently incomplete...
- ...but some interesting subset of sound inferences might be covered

[ter Horst, 2005],[Hogan *et al.*, 2008], see also: `http://www.polleres.net/presentations/20081029saor_ISWC_btriples_challenge.pdf`

Recommended Reading

- [ter Horst, 2005], RDF, RDFS and parts of OWL entailment in Rules.
- [Muñoz *et al.*, 2007], minimal sets of complete inference rules for parts of the RDFS vocabulary.

A bit more tough reading (specs), but also recommended:

- [Hayes, 2004, Sections 3 onwards], official RDF semantics specification.

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